JEPPIAAR ENGINEERING COLLEGE

Jeppiaar Nagar, Rajiv Gandhi Salai – 600 119

DEPARTMENT OFMECHANICAL ENGINEERING

QUESTION BANK

III SEMESTER

MA6351 Transforms and Partial Differential Equations

Regulation – 2013

SUBJECT : MA6351- TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

SEMESTER / YEAR: III / II

UNIT I - PARTIAL DIFFERENTIAL EQUATIONS

Formation of partial differential equations – Singular integrals -- Solutions of standard types of first order partial differential equations - Lagrange's linear equation -- Linear partial differential equations of second and higher order with constant coefficients of both homogeneous and non-homogeneous types.

PART- A

Q.No.	Question	Bloom's Taxonomy Level	Domain
1.	Form a partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $z = ax^2 + by^2$. **Solution** p=2ax, q=2by a= p/2x, b=q/2y therefore PDE is 2z=px+qy.	BTL -6	Creating
2.	Eliminate the arbitrary function from $z = f(y/x)$ and form the partial differential equation **MA6351 M/J 2014, N/D '14** **Solution:** px+qy=0	BTL -6	Creating
3.	Form the PDE from $(x-a)^2 + (y-b)^2 + z^2 = r^2$. **Solution** Differentiating the given equation w.r.t x &y, $z^2[p^2+q^2+1]=r^2$.	BTL -3	Applying
4.	Find the complete integral of p+q=pq. **Solution** p=a, q=b therefore $z = ax + \frac{a}{a-1}y + c$.	BTL- 6	Creating
5.	Form the partial differential equation by eliminating the arbitrary constants a, b from the relation $\log(az-1) = x + ay + b$. A/M'15 **Solution:** $\log(az-1) = x + ay + b$ Diff. p.w.r.t x&y, $\frac{ap}{az-1} = 1 - eqn1$ & $\frac{aq}{az-1} = a - eqn2$ $\frac{Eqn1}{Eqn2} \Rightarrow q = ap\ Sub\ in\ a(z-p) = 1 \Rightarrow q(z-p) = p$	BTL -6	Creating
6.	Form the PDE by eliminating the arbitrary constants a,b from the relation $z = ax^3 + by^3$. **MA6351 MAY/JUNE 2014** **Solution:** Differentiate w.r.t x and y $p = 3ax^2$, $q = 3by^2$ therefore 3z = px+qy.	BTL -6	Creating
7.	Form a p.d.e. by eliminating the arbitrary constants from $z = (2x^2+a)(3y-b)$. **Solution:** $p = 4x(3y-b)$, $q = 3(2x^2+a)$ $3y - b = p/4x$ $(2x^2+a) = q/3$. Therefore 12xz = pq.	BTL -6	Creating
8.	Form the partial differential equation by eliminating arbitrary function ϕ from $\phi(x^2 + y^2, z-xy) = 0$ [MA6351 M/J 2016] **Solution:** $u = x^2+y^2$ and v= z-xy. Then = 2x, $u_y = 2y$; $v_x = p - y$;	BTL -6	Creating

	v_y = q-x. $\begin{vmatrix} u_x & u_y \\ v_x & v_x \end{vmatrix} = 0 \Rightarrow 2xq - 2x^2 - 2yp + 2y^2 = 0$		
9.	Form the partial differential equation by eliminating arbitrary constants a and b from $(x-a)^2+(y-b)^2+z^2=1$ **<u>Solution</u>**: Differentiating the given equation w.r.t x &y, $z^2[p^2+q^2+1]=1$	BTL -6	Creating
10.	Solve $[D^3-8DD'^2-D^2D'+12D'^3]z = 0$ **[MA6351 M/J 2017]** **<u>Solution:</u>** The auxiliary equation is $m^3-m^2-8m+12=0$; m =2,2,-3 The solution is $z = f_1(y+x)+f_2(y+2x)+xf_3(y+2x)$. [MA6351 NOV/DEC 2014]	BTL -3	Applying
11.	Find the complete solution of $q = 2\ px$ **MA6351 APRIL/MAY 2015** **<u>Solution</u>** Find the complete solution of $q = 2\ px$ **<u>Solution:</u>** Let q= a then p = a/2x dz= pdx +qdy 2z = alogx+2ay+2b.	BTL -3	Applying

12.	Find the complete solution of p+q=1**[MA6351 NOV/DEC 2014]** **<u>Solution</u>** Complete integral is z = ax + F(a) y +c Put p = a, q = 1-a. Therefore z = px + (1-a) y +c	BTL -3	Applying
13.	Find the complete solution of $p^3 - q^3 = 0$ **MA6351 M/J 2016** **<u>Solution</u>** Complete integral is z = ax + F(a) y +c Put p = a, q = a. Therefore z = px + q y +c	BTL -3	Applying
14.	Solve $[D^3+DD'^2-D^2D'-D'^3]z = 0$ **<u>Solution</u>** The auxiliary equation is $m^3-m^2+m-1=0$ m =1,-i, i $\Rightarrow$ The solution is $z = f_1(y+x)+f_2(y+ix)+f_3(y-ix)$.	BTL -3	Applying
15.	Solve (D-1)(D-D'+1)z = 0. **<u>Solution</u>** $z = e^x f_1(y) + e^{-x} f_2(y+x)$	BTL -3	Applying
16.	Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 0.$ **<u>Solution:</u>** A.E: D[D-D'+1] = 0 h=0, h=k-1 $z = f_1(y) + e^{-x} f_2(y+x)$	BTL -3	Applying
17.	Solve $(D^4 - D'^4)z = 0$. **[MA6351 MAY/JUNE 2014]** **<u>Solution:</u>** A.E : $m^4-1=0$, m=±1, ±i. $Z=C.F=f_1(y+x)+f_2(y-x)+f_3(y+ix)+f_4(y-ix)$.	BTL -3	Applying
18.	Solve $(D^2 - DD'+D'-1)Z = 0$. **<u>Solution:</u>** The given equation can be written as $(D-1)(D-D'+1)Z = O$ $z = e^x f_1(y) + e^{-x} f_2(y+x)$	BTL -3	Applying
19.	Solve xdx + ydy = z.	BTL -3	Applying

	Solution The subsidiary equation is $\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$ $\frac{dx}{x}=\frac{dy}{y}\Rightarrow \log x=\log y+\log u$ $u=\frac{x}{y}$ Similarly $v=\frac{x}{z}$.		
20.	Form the p.d.e. by eliminating the arbitrary constants from z= ax +by +ab **Solution:** z= ax+by+ab p = a & q=b The required equation z= px+qy+pq.	BTL -3	Applying

PART – B			
1.(a)	Find the PDE of all planes which are at a constant distance ‘k’ units from the origin.	BTL -6	Creating
1. (b)	Find the singular integral of $z = px + qy + 1+ p^2 + q^2$	BTL -2	Understanding
2. (a)	Form the partial differential equation by eliminating arbitrary function Φ from $\Phi(x^2 + y^2 + z^2, ax + by + cz) = 0$	BTL -6	Creating
2.(b)	Find the singular integral of $z = px + qy + p^2 + pq + q^2$	BTL -2	Understanding
3. (a)	Form the partial differential equation by eliminating arbitrary functions *f* and *g* from z = x f(x/y) + *y g*(*x*)	BTL -6	Creating
3.(b)	Find the singular integral of $z = px + qy + \sqrt{1+ p^2 + q^2}$. MA6351 M/J 2016	BTL -3	Applying
4. (a)	Solve $(D^3-7DD'^2-6D'^3)z=\sin(x+2y)$.[MA6351 NOV/DEC 2014]	BTL -3	Applying
4.(b)	Form the partial differential equation by eliminating arbitrary function *f* and g from the relation $z = xf(x + t) + g(x + t)$	BTL -6	Creating
5. (a)	Solve $(D^2-2DD')z=x^3y+e^{2x-y}$. [MA6351 NOV/DEC 2014]	BTL -3	Applying
5.(b)	Solve x(y-z)p+y(z-x)q=z(x-y). [MA6351 NOV/DEC 2014]	BTL -3	Applying
6. (a)	Find the singular integral of $px+qy+p^2-q^2$. MA6351 NOV/DEC2014	BTL -2	Understanding
6.(b)	Find the general solution of $z = px + qy + p^2 + pq + q^2$. **M/J ‘17**	BTL -3	Applying
7. (a)	Find the complete solution of $z^2(p^2 + q^2 +1) = 1$	BTL -4	Analyzing

7. (b)	Find the general solution of $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$	BTL -2	Understanding
8. (a)	Find the general solution of $(D^2 + D'^2)z = x^2 y^2$	BTL -2	Understanding
8.(b)	Find the complete solution of $p^2 + x^2 y^2 q^2 = x^2 z^2$ A/M 2015	BTL -2	Understanding
9. (a)	Solve $(D^2 - 3DD' + 2D'^2)\, z = (2 + 4x)e^{x+2y}$	BTL -3	Applying
9.(b)	Obtain the complete solution of $z = px+qy+p^2-q^2$ MA6351 A/M 2015	BTL -2	Understanding
10.(a)	Solve $x(y^2 - z^2)\, p + y(z^2 - x^2)q = z(x^2 - y^2)$	BTL -3	Applying
10.(b)	Solve $(D^2 - 3DD' + 2D'^2)z = \sin(x + 5y)$	BTL -3	Applying
11(a)	Solve the Lagrange's equation $(x + 2z)\, p + (2xz - y)q = x^2 + y$	BTL -3	Applying
11(b)	Solve $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{2x+4y}$	BTL -3	Applying
12(a)	Solve $(D^2 + DD' - 6D'^2)z = y\cos x$	BTL -3	Applying
12(b)	Solve the partial differential equation $(x^2 - yz)p + (y^2 - xz)q = z^2 - xy$ MA6351 APRIL/MAY 2015, MA6351 M/J 2016	BTL -3	Applying
13(a)	Solve $(D^2 - DD' - 20D'^2)\, z = e^{5s+y} + \sin(4x - y)$.	BTL -3	Applying
13(b)	Solve $(2D^2 - DD' - D'^2 + 6D + 3D')z = xe^{y}$	BTL -3	Applying
14(a)	Solve $(D^2 - 2DD')z = x^3 y + e^{2x-y}$	BTL -3	Applying
14(b)	Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$	BTL -3	Applying
15(a)	Form the PDE by eliminating the arbitrary function from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. [MA6351 MAY/JUNE 2014]	BTL -6	Creating
15(b)	Solve the Lagrange's equation $(x+2z)p+(2xz-y) = x^2+y$.[MA6351 MAY/JUNE 2014]	BTL -3	Applying
16(a)	Solve $x^2p^2+y^2q^2 = z^2$. [MA6351 MAY/JUNE 2014]	BTL -3	Applying
16(b)	Solve $(D^2+DD'-6D'^2)z = y\cos x$. [MA6351 MAY/JUNE 2014]	BTL -3	Applying

UNIT II - FOURIER SERIES: Dirichlet's conditions – General Fourier series – Odd and even functions – Half range sine series – Half range cosine series – Complex form of Fourier series – Parseval's identity – Harmonic analysis.			
PART –A			
Q.No	**Question**	**Bloom's Taxonomy Level**	**Domain**
1.	State the Dirichlet's conditions for a function f(x) to be expanded as a Fourier series. **MA6351 MAY/JUNE 2014, A/M 2017** **Solution:** (i) f(x) is periodic, single valued and finite. (ii) f(x) has a finite number of discontinuities in any one period (iii) f(x) has a finite number of maxima and minima. (iv) f(x) and f'(x) are piecewise continuous.	**BTL -1**	Remembering
2.	Find the value of a_0 in the Fourier series expansion of $f(x)=e^x$ in $(0,2\pi)$. **[MA6351 MAY/JUNE 2014]** **Solution:** $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} e^x\,dx = 0$.	**BTL -1**	Remembering
3.	If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{cosnx}{n^2}\, in\ 0 < x < 2\pi$, then deduce that value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$. **[MA6351 NOV/DEC 2014]** **Solution**: Put x=0, $\sum_{n=1}^{\infty}\frac{1}{n^2} = 6$.	**BTL -1**	Remembering
4.	Does $f(x)$ = tanx posses a Fourier expansion? **Solution** No since tanx has infinite number of infinite discontinuous and not satisfying Dirichlet's condition.	**BTL -2**	Understanding
5.	Determine the value of a_n in the Fourier series expansion of $f(x) = x^3$ in $(-\pi, \pi)$. **Solution:** $a_n = 0$ since f(x) is an odd function	**BTL -5**	Evaluating
6.	Find the constant term in the expansion of $\cos^2 x$ as a Fourier series in the interval $(-\pi, \pi)$. **Solution:** $a_0 = 1$	**BTL -2**	Understanding
7.	If f(x) is an odd function defined in (-l, l). What are the values of a_0 and a_n? **Solution:** $a_n = 0 = a_0$	**BTL -2**	Understanding
8.	If the function f(x) = x in the interval 0<x<2 then find the constant term of the Fourier series expansion of the function f. **Solution:** $a_0 = 4\pi$	**BTL -2**	Understanding

9.	Expand f(x) =1 as a half range sine series in the interval $(0, \pi)$. **[MA6351 MAY/JUNE 2014]** **<u>Solution:</u>** The sine series of f(x) in $(0, \pi)$ is given by $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$ where $b_n = \frac{2}{\pi}\int_0^{\pi} \sin nx\,dx = -\frac{2}{n\pi}\left[\cos nx\right]_0^{\pi} = 0$ if n is even $= \frac{4}{n\pi}$ if n is odd $f(x) = \sum_{n=odd}^{\infty} \frac{4}{n\pi}\sin nx = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{(2n-1)}$.	**BTL -4**	Analyzing
10.	Find the value of the Fourier Series for f(x) = 0 -c<x<0 = 1 0<x<c at x = 0 **MA6351 M/J 2016** <u>Solution:</u> f(x) at x=0 is a discontinuous point in the middle. f(x) at x = 0 = $\frac{f(0-)+f(0+)}{2}$ $f(0-) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 0 = 0$ $f(0+) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} 1 = 1$ $\therefore$ f(x) at x = 0 → (0 + 1) / 2 = 1 / 2 = 0.5	**BTL -3**	Applying
11.	What is meant by Harmonic Analysis? <u>Solution</u>: The process of finding Euler constant for a tabular function is known as Harmonic Analysis.	**BTL -4**	Analyzing
12.	Find the constant term in the Fourier series corresponding to f (x) = $\cos^2 x$ expressed in the interval $(-\pi,\pi)$. <u>Solution:</u> Given $f(x) = \cos^2 x = \frac{1+\cos 2x}{2}$ W.K.T $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ To find $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\cos^2 x\,dx = \frac{2}{\pi}\int_0^{\pi}\frac{1+\cos 2x}{2}dx = \frac{1}{\pi}\left[x+\frac{\sin 2x}{2}\right]_0^{\pi}$ $= \frac{1}{\pi}[(\pi + 0) - (0+0)] = 1$.	**BTL -1**	Remembering
13.	Define Root Mean Square (or) R.M.S value of a function f(x) over the interval (a,b). <u>Solution:</u> The root mean square value of f(x) over the interval (a,b) is defined as	**BTL -3**	Applying

	R.M.S. $= \sqrt{\dfrac{\int_a^b [f(x)]^2\,dx}{b-a}}$.		
14.	Find the root mean square value of the function f(x) = x in the interval (0,l). Solution: The sine series of f(x) in (a,b) is given by R.M.S. $= \sqrt{\dfrac{\int_a^b [f(x)]^2\,dx}{b-a}} = \sqrt{\dfrac{\int_0^l [x]^2\,dx}{l-0}} = \dfrac{l}{\sqrt{3}}$.	**BTL -1**	Remembering
15.	If f(x) = 2x in the interval (0,4), then find the value of a_2 in the Fourier series expansion. Solution: $a_2 = \dfrac{2}{4}\int_0^4 2x\cos[\pi x]dx = 0.$	**BTL -5**	Evaluating
16.	To which value, the half range sine series corresponding to f(x) = x^2 expressed in the interval (0,5) converges at x = 5?. Solution: x = 2 is a point of discontinuity in the extremum. $\therefore [f(x)]_{x=5} = \dfrac{f(0)+f(5)}{2} = \dfrac{[0]+[25]}{2} = \dfrac{25}{2}$.	**BTL -6**	Creating
17.	If the Fourier Series corresponding to f(x) =x in the interval (0, 2 π) is $\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$ without finding the values of a_0, a_n , b_n find the value of $\dfrac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$. **[MA2211 APR/MAY 2011]** Solution: By Parseval's Theorem $\dfrac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2) = \dfrac{1}{\pi}\int_0^{2\Pi}[f(x)\}^2dx = \dfrac{1}{\pi}\int_0^{2\Pi}x^2dx = \dfrac{1}{\pi}\left[\dfrac{x^3}{3}\right]_0^{2\Pi}$ $= \dfrac{8}{3}\pi^2$	**BTL -4**	Analyzing
18.	Obtain the first term of the Fourier series for the function f(x) =x^2 , -π < x < π. Solution: Given f(x) =x^2 ,is an even functionin -π < x < π. Therefore, $a_o = \dfrac{2}{\pi}\int_0^{\pi} f(x)dx = \dfrac{2}{\pi}\int_0^{\pi} x^2dx = \dfrac{2}{\pi}\left[\dfrac{x^3}{3}\right]_0^{\pi} = \dfrac{2}{3}\pi^2.$	**BTL -1**	Remembering
19.	Find the co-efficient b_n of the Fourier series for the function f(x) = xsinx in (-2, 2). Solution: xsinx is an even function in (-2,2) . Therefore $b_n = 0$.	**BTL -6**	Creating

20.	Find the sum of the Fourier Series for f(x) = x 0<x<1 = 2 1<x<2 at x = 1. Solution: f(x) at x=1 is a discontinuous point in the middle. f(x) at x = 1 = $\frac{f(1-)+f(1+)}{2}$ f(1-) = $\lim_{h\to 0} f(1-h) = \lim_{h\to 0} 1-h = 1$ f(1+) = $\lim_{h\to 0} f(1+h) = \lim_{h\to 0} 2 = 2$ $\therefore$ f(x) at x = 1 ➔ (1 + 2) / 2 = 3 / 2 = 1.5	**BTL -3**	Applying
PART – B			

1.(a)	Obtain the Fourier's series of the function $f(x)=\begin{cases} x & for \quad 0<x<\pi \\ 2\pi - x & for \quad \pi<x<2\pi \end{cases}$. Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+....=\frac{\pi^2}{8}$ **MA6351A/M 2017**	**BTL -1**	Remembering
1.(b)	Find the Fourier's series of $f(x)=\lvert x\rvert$ in $-\pi<x<\pi$ And deduce that $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$	**BTL -1**	Remembering
2.(a)	Find the Fourier's series expansion of period $2l$ for $f(x)=(l-x)^2$ in the range $(0,2l)$. Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....=\frac{\pi^2}{6}$	**BTL -2**	Understanding
2.(b)	Find the Fourier series of periodicity 2π for f(x) $=x^2$ in $-\pi \le x \le \pi$. Hence deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots = \frac{\pi^4}{90}$.	**BTL -2**	Understanding
3.(a)	Find the Fourier series upto second harmonic for the following data: <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>f(x)</td><td>9</td><td>18</td><td>24</td><td>28</td><td>26</td><td>20</td></tr></table> **MA6351 APRIL/ MAY 2017**	**BTL -1**	Remembering

3.(b)	Find the Fourier series of $f(x) = 2x - x^2$ in the interval $0 < x < 2$	**BTL -1**	Remembering
4.(a)	Obtain the half range cosine series of the function $f(x)=\begin{cases} x \quad in \left(0,\frac{l}{2}\right) \\ l-x \left(\frac{l}{2},l\right) \end{cases}$.	**BTL -5**	Evaluating
4.(b)	Find the half range sine series of the function $f(x)=x(\pi-x)$ in the interval (0 , Л) .	**BTL -3**	Applying
5.(a)	Determine the Fourier series for the function $f(x)=\lvert\sin x\rvert \; in \; -\pi \le x \le \pi.$ MA6351 APRIL/ MAY 2015	**BTL -5**	Evaluating
5.(b)	Find the complex form of the Fourier series of $f(x) = e^{-ax}$ in (-l,l) **MA6351 APRIL/ MAY 2017**	**BTL -1**	Remembering
6.(a)	Find the Fourier series for $f(x) = x\sin x \;\; in \; (-\pi,\pi)$.	**BTL -2**	Remembering
6.(b)	Find the Fourier series expansion of $f(x) = x + x^2 \;\; -2 \le x \le 2.$	**BTL -2**	Remembering
7.(a)	Find the Fourier series for $f(x)=\begin{cases} x & (0,\pi/2) \\ \pi - x & (\pi/2,2\pi) \end{cases}$. MA6351 APRIL/ MAY 2015	**BTL -4**	Analyzing
7.(b)	Find the Fourier series of $f(x) = x + x^2$ in (-l, l) with period 2l.	**BTL -3**	Applying
8.(a)	Find the Fourier series as far as the second harmonic to represent the function f(x) with period 6, given in the following table.	**BTL -6**	Creating

X	**0**	**1**	**2**	**3**	**4**	**5**
f(x)	**9**	**18**	**24**	**28**	**26**	**20**

<table>
<tr><td>8.(b)</td><td>Find the complex form of the Fourier series of $f(x)=e^{-x}$ in $-1<x<1$. MA6351 APRIL/ MAY 2015</td><td>BTL -2</td><td>Remembering</td></tr>
<tr><td>9.(a)</td><td>Find the half range cosine series for the function $f(x)=x(\pi - x)$ in $0<x<\pi$. Deduce $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\dots=\frac{\pi^4}{90}$</td><td>BTL -2</td><td>Remembering</td></tr>
<tr><td>9.(b)</td><td>Obtain the Fourier series to represent the function $f(x)=|x|, -\pi<x<\pi$ and deduce $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$ (M/J 2012)</td><td>BTL -3</td><td>Applying</td></tr>
<tr><td>10.(a)</td><td>Find the half range sine series of $f(x) = lx-x^2$ in $(0,l)$ (N/D 2013)</td><td>BTL -1</td><td>Remembering</td></tr>
<tr><td>10.(b)</td><td>Obtain the Fourier cosine series expansion of $f(x) = x$ in $0<x<4$. Hence deduce the value of $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\dots$</td><td>BTL -1</td><td>Remembering</td></tr>
<tr><td>11.(a)</td><td>By using Cosine series show that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\dots=\frac{\pi^4}{96}$ for $f(x) = x$ in $0<x<\pi$</td><td>BTL -4</td><td>Analyzing</td></tr>
<tr><td>11.(b)</td><td>Find the Fourier cosine series up to third harmonic to represent the function given by the following data: MA6351 M/J 2016
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>Y</td><td>4</td><td>8</td><td>15</td><td>7</td><td>6</td><td>2</td></tr>
</table></td><td>BTL -6</td><td>Creating</td></tr>
<tr><td>12.(a)</td><td>Show that the complex form of Fourier series for the function $f(x)=e^{ax}$ $(-\pi,\pi)$)</td><td>BTL -1</td><td>Remembering</td></tr>
<tr><td>12.(b)</td><td>Find the complex form of the Fourier series of $f(x)=e^{-x}$ in $-1<x<1$. (N/D 2009)</td><td>BTL -4</td><td>Analyzing</td></tr>
<tr><td>13.</td><td>Calculate the first 3 harmonics of the Fourier of f(x) from the following data
<table>
<tr><td>x</td><td>0</td><td>30</td><td>60</td><td>90</td><td>120</td><td>150</td><td>180</td><td>210</td><td>240</td><td>270</td><td>300</td><td>330</td><td>360</td></tr>
<tr><td>f(x)</td><td>1.8</td><td>1.1</td><td>0.3</td><td>0.16</td><td>0.5</td><td>1.3</td><td>2.16</td><td>1.25</td><td>1.3</td><td>1.52</td><td>1.76</td><td>2</td><td>1.8</td></tr>
</table></td><td>BTL -6</td><td>Creating</td></tr>
</table>

14.(a)	Find the complex form of the Fourier series of $f(x) = e^{-s}$ in $-1 < x < 1$.	**BTL -4**	Analyzing
14.(b)	Find the Fourier series up to the second harmonic from the following table.	**BTL -6**	Creating

x	0	1	2	3	4	5
f(x)	9	18	24	28	26	20

UNIT – III APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Solution of one dimensional wave equation-One dimensional heat equation-Steady state solution of two dimensional heat equation-Fourier series solutions in Cartesian coordinates .

Textbook : Grewal. B.S., "Higher Engineering Mathematics", 42nd Edition, Khanna Publishers, Delhi, 2012.

PART – A

Q.No	Questions	BT Level	Competence	PO
1	What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation. Solution: The correct solution of one dimensional wave equation is of periodic in nature. But the solution of heat equation is not periodic in nature. **(A.U.N/D 2017, N/D 2011,2012, M/J 2013)**	**BTL-4**	**Analyzing**	**PO1**
2	In steady state conditions derive the solution of one dimensional heat flow equations. [Nov / Dec 2005] Solution: one dimensional heat flow equation is $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ ……….(1) When the steady state conditions exists, put $\frac{\partial u}{\partial t} = 0$ Then (1) becomes, $\frac{\partial^2 u}{\partial x^2} = 0$. Solving, we get u(x)=ax+b. a and b are arbitrary constants.	**BTL-2**	**Understanding**	**PO1,PO2,PO3**

3	What are the possible solution of one dimensional wave equation. Solution: The possible solutions are (i) y(x,t)=($A_1e^{px}+A_2e^{-px})(A_3e^{pat}+A_4e^{-pat})$ (ii) y(x,t)=($B_1\cos px+B_2\sin px)(B_3\cos pat+B_4\sin pat)$ (iii) y(x,t)=($C_1x+C_2)(C_3t+C_4)$. **(A.U. M/J 2014)**	**BTL-1**	**Remembering**	**PO1,PO2**
4	Classify the P.D.E $3u_{xx}+4u_{yy}+3u_y-2u_x=0$. Solution: $B^2-4AC=16-4(3)(0)=16>0$. It is hyperbolic. **(A.U M/J 2008)**	**BTL-1**	**Remembering**	**PO1**
5	The ends A and B of a rod of length 10cm long have their temperatures kept at $20^o C and 70^o C$.Find the Steady state temperature distribution of the rod. Solution: The initial temperature distribution is u(x,0)= $\frac{b-a}{l}x+a$.Here $a=20^o C, b=70^o C, l=10cm.$ $\therefore u(x,t)=\frac{70-20}{10}x+20=5x+20.0<x<10.$ **(A.U M/J 2008, A.U N/D 2012)**	**BTL-1**	**Remembering**	**PO1**
6	Classify the PDE $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial x\partial y}+4\frac{\partial^2 u}{\partial y^2}-12\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+7u=x^2+y^2.$ Solution: The given PDE is $u_{xx}+4u_{xy}+4u_{yy}-12u_x+u_y+7u=x^2+y^2$.A=1;B=4;C=4. $B^2-4AC=16-16=0.$ $\therefore$ The given PDE is parabolic. **(A.U M/J 2009)**	**BTL-3**	**Applying**	**PO1**
7	Write down the one dimensional heat equation. Solution: The one dimensional heat equation is $\frac{\partial^2 u}{\partial x^2}=\frac{1}{\alpha^2}\frac{\partial u}{\partial t}$. **(A.U M/J 2010,N/D 2012)**	**BTL-1**	**Remembering**	**PO1,PO2**
8	Write down the possible solutions of one dimensional heat flow equation. Solution: The various possible solutions of one dimensional heat equation are (i)u(x,t)= $(Ae^{px}+Be^{-px})e^{\alpha^2p^2t}$ (ii) ($A\cos px+B\sin px)e^{-\alpha^2p^2t}$ (iii) u(x,t)=(Ax+B).	**BTL-1**	**Remembering**	**PO1,PO2**

	(A.U M/J 2009,A.U N/D 2014)			
9	Write the one dimensional wave equation with initial and boundary conditions in which the initial position of the string is f(x) and the initial velocity imparted at each point is g(x). Solution: The wave equation is $\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2}$. The boundary conditions are (i) y(0,t)=0, ∀ t>0 (ii) y(0,t)=0, $\forall t > 0$ (iii) $\frac{\partial y}{\partial t}(x,0) = g(x), 0 < x < l.$ (iv) y(x,0)=f(x),0<x<l. **(A.U N/D 2007,M/J 2012,2016)**	**BTL-3**	**Applying**	**PO1**
10	Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$ Solution: Given $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$ $\alpha^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ $Here A = \alpha^2; B = 0; C = 0.$ $\therefore B^2 - 4AC = 0 - 4(\alpha^2)(0) = 0.$ **(A.U M/J 2007,M/J2013 ,M/J 2016)**	**BTL-1**	**Remembering**	**PO1**
11	State the two dimensional Laplace equation? Solution : $U_{xx} + U_{yy} = 0$ (**A.U., N/D 2011,2012, M/J 2014)**	**BTL-1**	**Remembering**	**PO1**
12	In an one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ what does the constant stands for ? Solution : α^2 is called the diffusivity of the material of the body through which the heat flows. If ρ be the density, α the specific heat and k thermal conductivity of the material, we have the relation $k/\rho\alpha = c2$. **(A.U M/J 2013)**	**BTL-1**	**Remembering**	**PO1**
13	What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation. Solution: Solution of the one dimensional wave equation is of periodic in nature. But Solution of the one dimensional heat equation is not of periodic in nature.	**BTL-1**	**Remembering**	**PO1,PO2,PO5**

	(A.U A/M 2016)			
14	In the wave equation $\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2}$, What does α^2 stands for ? Solution : $\alpha^2 = \frac{Tension}{MassperUnitlength}$ **(A.U M/J 2014)**	**BTL-1**	**Remembering**	**PO1**
15	In 2D heat equation or Laplace equation ,What is the basic assumption? Solution : When the heat flow is along curves instead of straight lines,the curves lying in parallel planes the flow is called two dimensional **(A.U M/J 2016)**	**BTL-4**	**Analyzing**	**PO1**
16	Define steady state condition on heat flow. Solution: Steady state condition in heat flow means that the temp at any point in the body does not vary with time. That is, it is independent of t, the time. **(MA2211 A.U M/J 2013)**	**BTL-1**	**Remembering**	**PO1**
17	Write the solution of one dimensional heat flow equation , when the time derivative is absent. Solution : When time derivative is absent the heat flow equation is $U_{xx} = 0$ (**A.U N/D 2009, N/D 2015)**	**BTL-2**	**Understanding**	**PO2**
18	If the solution of one dimensional heat flow equation depends neither on Fourier cosine series nor on Fourier sine series , what would have been the nature of the end conditions? Solution :. One end should be thermally insulated and the other end is at zero temperature. (**A.U M/J 2017)**	**BTL-1**	**Remembering**	**PO1**
19	State any two laws which are assumed to derive one dimensional heat equation? Solution : (i)The sides of the bar are insulated so that the loss or gain of heat from the sides by conduction or radiation is negligible. (ii)The same amount of heat is applied at all points of the face **(A.U M/J 2013)**	**BTL-1**	**Remembering**	**PO1,PO2**
20	What are the assumptions made before deriving the one dimensional heat equation? **Solution : (i)**Heat flows from a higher to lower temperature. (ii)The amount of heat required to produce a given temperature change in a body is proportional to the mass of the body and to the temperature change. (iii)The rate at which heat flows through an area is	**BTL-1**	**Remembering**	**PO1,PO2**

	proportional to the area and to the temperature gradient normal to the area. **(A.U M/J 2014)**			
21	Write down the two dimensional heat equation both in transient and steady states. Solution : Transient state: $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ Steady state: : $U_{xx} + U_{yy} = 0$ **(A.U M/J 2013)**			
PART-B				
1	A uniform string is stretched and fastened to two points $'l'$ apart. Motion is started by displacing the string into the form of the curve $y = kx(l - x)$ and then releasing it from this position at time $t = 0$. Find the displacement of the point of the string at a distance x from one end at time t. **(A.U.N/D 2017, N/D 2015,2012, M/J 2013)**	**BTL-4**	**Analyzing**	**PO1,PO2,PO5**
2	A tightly stretched string of length l has its ends fastened at $x = 0$ and $x = l$. The midpoint of the string is then taken to a height h and then released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time.	**BTL-4**	**Analyzing**	**PO1,PO2,PO5**
3	A tightly stretched string of length $2l$ is fastened at both ends. The midpoint of the string is displaced by a distance $'b'$ transversely and the string is released from rest in this position. (Find the lateral displacement of a point of the string at time $'t'$ from the instant of release) Find an expression for the transverse displacement of the string at any time during the subsequent motion (**A.U. N/D 2011, M/J 2014)**	**BTL-5**	**Evaluating**	**PO1,PO2,PO5**
4	A tightly stretched string of length l is initially at rest in equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial y}\right)_{t=0} = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement at any time $'t'$. (**A.U N/D 2014)**	**BTL-5**	**Evaluating**	**PO1,PO2,PO5**
5	A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial	**BTL-2**	**Understanding**	**PO1,PO2**

	velocities v where $v = \begin{cases} \frac{cx}{l} & 0 < x < l \\ \frac{c}{l}(2l - x) & l < x < 2l \end{cases}$, x being the distance from one end point. Find the displacement of the string at any subsequent time. *(***A.U.N/D 2008***)*.			
6	A rod 30cm long has its ends A and B kept at $20^{\circ}c$ and $80^{\circ}c$ respectively until steady state conditions prevails. The temperature at each end is then suddenely reduced to $0^{\circ}c$ and kept so. Find the resulting temperature function $u(x,t)$ taking $x = 0$ at A.*(Nov./Dec. 2009).* **(A.U.N/D 2011, M/J 2014)**	**BTL-2**	**Understanding**	**PO1,PO2**
7	A rod of length l has its ends A and B kept at $0^{\circ}c$ and $120^{\circ}c$ respectively until steady state conditions prevail. If the temperature at B is reduced to $0^{\circ}c$ and so while that of A is maintained, find the temperature distribution of the rod. **(A.U M/J 2012,16)**	**BTL-5**	**Evaluating**	**PO1,PO2,PO5**
8	An infinitely long rectangular plate with insulated surface is 10 cm wide. The two long edges and one short edge are kept at zero temperature, while the other short edge x = 0 is kept at temperature given by $u = \begin{cases} 20y, & 0 \le y \le 5 \\ 20(10 - y), & 5 \le y \le 10 \end{cases}$. **(A.U M/J 2011,N/D 2012,M/J 2016)**	**BTL-5**	**Evaluating**	**PO1,PO2,PO5**
9	A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form y = k(lx-x^2) from which it is released at time t = 0. Find the displacement of any point on the string at a distance x from one end at time t. **(A.U M/J 2015,2016)**	**BTL-5**	**Evaluating**	**PO1,PO2,PO5**
10	A square plate is bounded by the lines x = 0, x = a, y = 0 and y = b. Its surfaces are insulated and the temperature along y = b is kept at 100° C. Find the steady-state temperature at any point in the plate. [A.U. N/D 2014]	**BTL-5**	**Evaluating**	**PO1,PO2,PO5**
11	A tightly stretched string with fixed end points x = 0 and x = l is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find	**BTL-2**	**Understanding**	**PO1,PO2**

	the displacement at any time $'t'$. **(A.U N/D 2012)**			

UNIT – IV FOURIER TRANSFORM

Fourier integral theorem (without proof) – Fourier transform pair –Sine and Cosine transforms- Properties – Transforms of simple functions – Convolution theorem – Parseval's identity.
Textbook : Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition,Khanna Publishers, New Delhi, 2007.

PART – A

CO Mapping : C214.2

Q.No	Questions	BT Level	Competence	PO
1	Prove that $F[f(x-a)] = e^{ias}F(s)$ **<u>Proof:</u>** $F(f(x)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$ $F(f(x-a)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x-a)e^{isx}dx, \quad put\, t = x-a; dt = dx$ $x \to \pm\infty \Rightarrow t \to \pm\infty$ $F(f(x-a)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{is(t+a)}dt = e^{isa}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{ist}dt = e^{isa}F(s).$ **(A.U.N/D , N/D 2011,2012, M/J 2013)**	**BTL-4**	**Analyzing**	**PO1**
2	Prove that $F(f(x)\cos ax) = \frac{1}{2}[F(s+a)+F(s-a)]$. [MA2211 APR/MAY2011] **<u>Proof:</u>**	**BTL-1**	**Remembering**	**PO1,PO2**

	$F\left(f(x)\cos ax\right)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cos ax\, e^{isx}dx$ $=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\frac{e^{iax}+e^{-iax}}{2}e^{isx}dx$ $=\frac{1}{2}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{i(s+a)x}dx+\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{i(s-a)x}dx\right)$ $=\frac{1}{2}\left[F(s+a)+F(s-a)\right].$ **(A.U.N/D 2017, N/D 2015,2012, M/J 2011)**			
3	Prove that $F_c\left(f(x)\sin ax\right)=\frac{1}{2}\left[F_s(s+a)+F_s(s-a)\right]$ **Proof:** $F_c\left(f(x)\cos ax\right)=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty} f(x)\sin ax\cos sx\,dx$ $=\frac{1}{2}\sqrt{\frac{2}{\pi}}\int_{0}^{\infty} f(x)\left(\sin(s+a)x+\sin(s-a)x\right)dx$ $=\frac{1}{2}\left(\sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} f(x)\sin(s+a)x\,dx+\sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} f(x)\sin(s-a)x\,dx\right)$ $=\frac{1}{2}\left[F_s(s+a)+F_s(s-a)\right].$ **(A.U M/J 2011,2008)**	**BTL-2**	**Understanding**	**PO1,PO2**
4	Find the Fourier sine transform of e^{-x}, $x>0$. **Solution:** $F_s\left(f(x)\right)=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty} f(x)\sin sx\,dx=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty} e^{-x}\sin sx\,dx$ $=\sqrt{\frac{2}{\pi}}\left[\frac{e^{-x}}{1+s^2}\left(-\sin sx-s\cos sx\right)\right]_0^{\infty}=\sqrt{\frac{2}{\pi}}\frac{s}{1+s^2}$ (A.U.N/D 2010)	**BTL-4**	**Analyzing**	**PO2**
5	Write the Fourier transform pair. **Proof:**	**BTL-1**	**Remembering**	**PO1**

	$$F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$$ $$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)\,e^{-isx}ds$$ **(A.U N/D 2009,N/D 2012,M/J 2015,2016)**			
6	Find the Fourier sine transform of $\frac{1}{x}$. **Solution:** $$F_s(f(x))=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin sx dx=\sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{1}{x}\sin sx dx$$ $put\,sx=\theta;\ sdx=d\theta;$ $$=\sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{\sin\theta}{\theta}d\theta=\sqrt{\frac{2}{\pi}}\frac{\pi}{2}=\sqrt{\frac{\pi}{2}}.$$ **(A.U M/J 2009,N/D 2012,M/J 2015,2016)**	**BTL-2**	**Understanding**	**PO1**
7	Find the Fourier cosine transform of $f(ax)$. **Solution:** $$F_c(f(ax))=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(ax)\cos sx dx$$ $put\ \ t=ax;\ dt=adx$ $$=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\cos\left(\frac{st}{a}\right)\frac{dt}{a}=\frac{1}{a}F_c\left(\frac{s}{a}\right).$$ **(A.U M/J 2013)**	**BTL-2**	**Understanding**	**PO1,PO2**
8	Find the Fourier Cosine transform of e^{-ax}. **Solution:** $$F_c[e^{-ax}]=\sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-ax}\cos sx dx=\sqrt{\frac{2}{\pi}}\left[\frac{e^{-ax}}{a^2+s^2}(-a\cos sx+s\sin sx)\right]_0^{\infty}$$ $$=\sqrt{\frac{2}{\pi}}\frac{a}{a^2+s^2}.$$ **(A.U M/J 2012 N/D 2015,N/D 2009)**	**BTL-1**	**Remembering**	**PO1**
9	Find the Fourier transform of $f(x)=\begin{cases} e^{ikx}, & a<x<b \\ 0, & x<a, x>b \end{cases}$ **Solution:**	**BTL-1**	**Remembering**	**PO1**

	$F[f(x)]=\frac{1}{\sqrt{2\pi}}\int_a^b e^{ikx}e^{isx}dx=\frac{1}{\sqrt{2\pi}}\int_a^b e^{i(s+k)x}dx=\frac{1}{\sqrt{2\pi}}\left[\frac{e^{i(s+k)x}}{i(s+k)}\right]_a^b$ $=\frac{1}{\sqrt{2\pi}}\left[\frac{e^{i(s+k)b}-e^{i(s+k)a}}{i(s+k)}\right].$ **(A.U M/J 2016,N/D 2012,N/D 2009)**			
10	State convolution theorem. Solution : If F(s) and G(s) are fourier transforms of f(x) and g(x) respectively then the fourier transform of the convolutions of f(x) and g(x) is the product of their fourier transform. **(A.U N/D 2012,M/J 2016)**	**BTL-1**	**Remembering**	**PO1**
11	Write the Fourier cosine transform pair? Solution : $F_c(s)=\frac{2}{\sqrt{\pi}}\int_0^\infty f(x)\cos sx dx$ $f(x)=\frac{\sqrt{2}}{\sqrt{\pi}}\int_0^\infty F_c(f(x)\cos sx ds$ **(A.U N/D 2011,N/D 2014)**	**BTL-2**	**Understanding**	**PO1,PO2**
12	Write Fourier sine transform and its inversion formula? Solution : $F_s(s)=\frac{2}{\sqrt{\pi}}\int_0^\infty f(x)\sin sx dx$ $f(x)=\frac{\sqrt{2}}{\sqrt{\pi}}\int_0^\infty F_s(f(x)\sin sx ds$	**BTL-4**	**Analyzing**	**PO1**
13	State the modulation theorem in Fourier transform . Solution : If F(s) is the Fourier transform of f(x) , then F[f(x) cos ax] = 1/2 [F (s+a) +F(s-a). (A.U.N/D 2014)	**BTL-4**	**Analyzing**	**PO1,PO2**
14	State the Parsevals identity on Fourier transform. Solution : If F(s) is the Fourier transform of f(x), then $\int_{-\infty}^{\infty}\lvert f(x)\rvert^2 dx = \int_{-\infty}^{\infty}\lvert F(s)\rvert^2 ds$ **(A.U N/D 2014,M/J 2016)**	**BTL-4**	**Analyzing**	**PO1**
15	State Fourier Integral theorem **.** **Solution :** If f(x) is piecewise continuously differentiable & absolutely integrable in (-∞ , ∞) then f(x) = $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t)e^{is(x-t)}dtds$ This is known as Fourier integral theorem **(A.U M/J 2014)**	**BTL-1**	**Remembering**	**PO1**
16	Define self-reciprocal with respect to Fourier Transform. Solution: If a transformation of a function f(x) is equal to f(s) then the function f(x) is called self-reciprocal	**BTL-4**	**Analyzing**	**PO1**

	[A.U. N/D 2013]			
PART – B				
1	Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & \lvert x\rvert \le a \\ 0, & \lvert x\rvert \phi\ a \end{cases}$. Hence evaluate $\int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\left(\frac{s}{2}\right)dx.$ **(A.U M/J 2011,A/M 2012,N/D 2015)**	**BTL-5**	**Evaluating**	**PO1,PO2, PO3,PO5**
2	Find the Fourier cosine transform of $f(x)=e^{-ax}, a>0$ and $g(x)=e^{-bx}, b>0.$ Hence evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+9)}.$ (A.UA/M 2009)	**BTL-4**	**Analyzing**	**PO1,PO2**
3	Find the Fourier Transform of f(x) given by $f(x)=\begin{cases} a-\lvert x\rvert, & \lvert x\rvert \le a \\ 0, & \lvert x\rvert \phi\ a \end{cases}$. Hence show that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}\ and \int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}.$ **(A.U.N/D 2017, N/D 2011,2012, M/J 2013)**	**BTL-5**	**Evaluating**	**PO1,PO2, PO5,PO12**
4	Find the Fourier transform of $f(x)=\begin{cases} 1, \ for\ \lvert x\rvert \le a \\ 0, \ for\ \lvert x\rvert \phi\ a \ \phi\ 0 \end{cases}$ and using Parseval's identity prove that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}.$ **(A.U.N/D 2017, N/D 2011,2014, M/J 2013)**	**BTL-4**	**Analyzing**	**PO1,PO2, PO5,PO12**
5	Find the Fourier sine and cosine transform of e^{-ax} and hence find the Fourier sine transform of $\frac{x}{x^2+a^2}$ and Fourier cosine transform of $\frac{1}{x^2+a^2}$. (A.U. N/D 2011)	**BTL-5**	**Evaluating**	**PO1,PO2, PO5,PO12**
6	Find the Fourier cosine transform of e^{-x^2}. **(A.U M/J 2016)**	**BTL-4**	**Analyzing**	**PO1,PO2, PO5,PO12**
7	Prove that $\frac{1}{\sqrt{x}}$ is self reciprocal under Fourier sine	**BTL-5**	**Evaluating**	**PO1,PO2, PO5,PO12**

	and cosine transforms. **(A.U N/D 2015,M/J 2014)**			
8	Evaluate $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier transforms(MA1201 N/D 2005, M/J 2014,N/D2016)	**BTL-5**	**Evaluating**	**PO1,PO2, PO5,PO12**
9	By finding the Fourier cosine transform of $f(x)=e^{-ax}\,(a \phi 0)$ and using Parseval's identity for cosine transform evaluate $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$. **(A.U A/M 2016)**	**BTL-3**	**Applying**	**PO1,PO2, PO5,PO12**
10	If $F_c(s)$ *and* $G_c(s)$ are the Fourier cosine transform of f(x) and g(x) respectively, then prove that $\int_0^\infty f(x)g(x)dx = \int_0^\infty F_c(s)G_c(s)ds.$ (A.U N/D 2008,M/J 2015)	**BTL-3**	**Applying**	**PO1,PO2, PO5,PO12**
11.	Find the Fourier sine transform of $f(x)=\begin{cases} x, & 0 \pi x \pi 1 \\ 2-x, & 1 \pi x \pi 2 \\ 0, & x \phi 2. \end{cases}$ **(A.U.N/D 2016, N/D 2011,2012, M/J 2014)**	**BTL-5**	**Evaluating**	**PO1,PO2, PO5,PO12**
12.	If $F_c(f(x)) = F_c(s)$, prove that $F_c(F_c(x)) = f(s)$. **(A.U M/J 2017)**	**BTL-3**	**Applying**	**PO1,PO2, PO5,PO12**
13	Use transform method to evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$	**BTL-3**	**Applying**	**PO1,PO2, PO5,PO12**

UNIT-V Z -TRANSFORMS AND DIFFERENCE EQUATIONS

Z-transforms - Elementary properties – Inverse Z-transform – Convolution theorem -Formation of difference equations – Solution of difference equations using Z-transform.

PART – A

CO Mapping :

Q.No	**Questions**	**BT Level**	**Competence**	**PO**
1.	Define the unit step sequence. Write its Z- transform. Soln: It is defined as U(k) : {1 ,1 , 1, ………) = $\begin{cases} 1, & k > 0 \\ 0, & k < 0 \end{cases}$	**BTL -1**	**Remembering**	**PO1**

	Hence $Z[u(k)] = 1 + 1/z + 1/z^2 + \ldots + = \frac{1}{1-1/z} = \frac{z}{z-1}$ **(A.U.N/D 2017, N/D 2010,2012, M/J 2013)**			
2.	Form adifference equation by eliminating the arbitrary constant A from $y_n = A.3^n$ Soln: $y_n = A.3^n$,$y_{n+1} = A.3^{n+1} = 3A\ 3^n = 3y_n$ Hence $y_{n+1} - 3y_n\ 0$ **(A.U N/D 2010,M/J 2012,2014)**	**BTL -1**	**Understanding**	**PO1**
3.	Find the Z transform of $\sin \frac{n n\pi}{2}$ Soln: We know that , $z[\sin n\theta] = \frac{Z\ sinn\theta}{z2-2zcos\theta+1}$ Put $\theta = \pi/2$ $z[\sin \frac{n\pi}{2}] = \frac{z \sin n\pi/2}{z2-\frac{2zcos\pi}{2}+1} = \frac{z}{z2+1}$ **(A.U.A/M 2010, M/J 2012)**	**BTL -5**	**Understanding**	**PO1**
4.	Find Z(n). Soln: $Z(n) = \frac{z}{(z-1)2}$ **(A.U M/J 2011)**	**BTL -1**	**Remembering**	**PO1**
5.	Express Z{ f(n+1)} in terms of f(z) Soln: Z{ f(n+1)} = zf(z) – zf(0) **(A.U M/J 2011)**	**BTL -1**	**Remembering**	**PO1**
6.	Find the value of z{f(n)] when $f(n) = na^n$ Soln: $z(na^n) = \frac{az}{(z-a)}$ **(A.U M/J 2009)**	**BTL -1**	**Understanding**	**PO2,PO5**
7.	Find $z[e^{-iat}]$ using Z transform. Soln. By shifting property , $z[e^{-iat}] = ze^{iaT}/\ ze^{iaT}{}_{-1}$ **(A.U M/J 2011,N/D 2012)**	**BTL -1**	**Remembering**	**PO1**
8.	Find the Z transform of $a^n/n!$. Soln: $z[a^n/n!] = e^{a/z}$ (By definition) **(A.U M/J 2012,N/D 2014)**	**BTL -1**	**Understanding**	**PO1**
9.	State initial value theorem in Z-transform. Solution : If f(t) = F(z) then $\lim_{t\to 0} f(t) = \lim_{z\to\infty} F(z)$. (A.U.N/D 2015,2013)	**BTL -1**	**Understanding**	**PO1**
10.	State final value theorem in Z-transform. Solution : If f(t) = F(z) then $\lim_{t\to\infty} f(t) = \lim_{z\to 0} F(z)$. State Euler formula. **(A.U M/J 2013)**	**BTL -1**	**Understanding**	**PO1**
11.	State Convolution theorem on Z-transform. Solution : If X(z)) and Y(z) are Z- transforms of x(n) and y(n) respectively then the Z- transform of the convolutions of x(n) and y(n) is the product of their Z- transform. **(A.U M/J 2012,N/D 2014)**	**BTL -1**	**Understanding**	**PO1**
12.	Define Z-tranforms of f(t). Solution : Z-transform for discrete values of t : If f(t) is a	**BTL -1**	**Understanding**	**PO1**

	function defined for discrete values of t where t=nT , n=0,1,2,...T being the sampling period then Z{f(t)} = F(Z) = $\sum_{n=0}^{\infty} f(nT)Z^{-n}$			
13.	Define Z- transform of the sequence. Solution : Let {x(n)} be a sequence defined for all integers then its Z-transform is defined to be Z{x(n)} = X(Z) = $\sum_{n=0}^{\infty} x(n)Z^{-n}$	**BTL -4**	**Analyzing**	**PO2**
14.	State first shifting theorem. Solution : If Z{f(t)} = F(Z) then Z{(e^{-at} f(t) } = F(ze^{at})	**BTL -2**	**Remembering**	**PO5**
15.	Find the Z-Transform of cos nθ and sin nθ? Solution : Z(cos nθ) = $\frac{z(z-\cos\theta)}{(z-\cos\theta)^2+\sin^2\theta}$ Z(sinnθ) = $\frac{z\sin\theta}{(z-\cos\theta)^2+\sin^2\theta}$ **(A.U N/D 2017)**	**BTL -2**	**Remembering**	**PO5**
16.	Find the Z-transform of unit step sequence. Solution: $u(n)=1$ for n ≥ 0 $u(n)=0$ for n < 0. Now Z[u(n)] = $\frac{z}{z-1}$	**BTL -1**	**Remembering**	**PO1**
17.	Find the Z-transform of unit sample sequence. Solution: $\delta(n)=1$ for n = 0 $\delta(n)=0$ for n > 0. Now Z[δ (n)] = 1	**BTL -1**	**Understanding**	**PO1**
18.	Form a difference equation by eliminating arbitrary constant from $u_n = a.2^{n+1}$. Solution : Given , $u_n = a.2^{n+1}$ $u_{n+1} = a.2^{n+2}$ Eliminating the constant a, we get $\begin{vmatrix} u_n & 1 \\ u_{n+1} & 2 \end{vmatrix} = 0$ We get $2u_n - u_{n+1} = 0$	**BTL -1**	**Understanding**	**PO1**
19.	Form the difference equation from $y_n = a + b.3^n$ Solution: Given , $y_n = a + b.3^n$ $y_{n+1} = a + b.3^{n+1}$ $= a+3b.3^n$ $y_{n+2} = a + b.3^{n+2}$ $= a+9b.3^n$	**BTL -1**	**Understanding**	**PO1**

	Eliminating a and b we get, $\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 1 & 3 \\ y_{n+2} & 1 & 9 \end{vmatrix} = 0$ We get $y_{n+2} - 4y_{n+1} + 3y_n = 0$			
20.	Find $Z[\frac{a^n}{n!}]$ Solution : $Z[\frac{a^n}{n!}] = e^{\frac{a}{z}}$			
PART-B				
1.	Find the Z-transform of cos nθ and sin nθ. Hence deduce the Z-transform of cos (n + 1)θ and a^n sin nθ (A.U.N/D 2012,2015,M/J 2016)	**BTL -1**	**Remembering**	**PO1,PO2,PO5**
2	Use residue theorem find $Z^{-1}\left(\frac{z\ (z+1)}{(z-3)^3}\right)$	**BTL -3**	**Applying**	**PO1,PO2,PO5**
3	Solve $y_{n+2} - 5y_{n+1} + 6y_n = 6^n$, $y_0 = 1$, $y_1 = 0$ (A.U.N/D 2012,2016)	**BTL -1**	**Remembering**	**PO1,PO2,PO5**
4	Solve using Z-Transform $u_{n+2} + 6u_{n+1} + 9u_n = 2^n;\quad given\ u_0 = u_1 = 0$ [NOV/DEC 2010]	**BTL -1**	**Remembering**	**PO1,PO2,PO5**
5	Using convolution theorem find the inverse Z transform of [APRIL/MAY 2010] $\left(\frac{z}{z-4}\right)^3$ (A.U.M/J 2010, N/D2014)	**BTL -2**	**Understanding**	**PO1,PO2,PO5**
6	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = 0$, $y_1 = 0$ (A.U.N/D 2014,2017)	**BTL -1**	**Remembering**	**PO1,PO2,PO5**
7	Using convolution theorem find $Z^{-1}\left(\frac{z^2}{(z-4)(z-3)}\right)$ (A.U.N/D 2014,2016, M/J2013)	**BTL -1**	**Remembering**	**PO1,PO2,PO5**
8	Fnd the inverse Z –transform of $\frac{Z^3-20Z}{(Z-2)^3(Z-4)}$ (A.U.N/D 2009)	**BTL -3**	**Applying**	**PO1,PO2,PO12**
9	Find $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z+1)}\right)$ (A.U.N/D 2015,2013)	**BTL -3**	**Applying**	**PO1,PO2,PO12**

10	State and Prove Convolution theorem (A.U.N/D 2016,2011)	**BTL -3**	**Applying**	**PO1,PO2,PO12**
11	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = 0$, $y_1 = 0$ (A.U.A/M 2008,2010,N/D2014)	**BTL -5**	**Evaluating**	**PO1,PO2,PO5**
12	Prove that Z $\left(\frac{1}{n}\right)$ = log $\left(\frac{z}{z-1}\right)$	**BTL -3**	**Applying**	**PO1,PO2,PO12**
13	Using convolution theorem evaluate inverse Z-transform of $\left[\frac{z^2}{(z-1)(z-3)}\right]$ (A.U.N/D 2010,2016)	**BTL -1**	**Remembering**	**PO1,PO2,PO5**