JEPPIAAR ENGINEERING COLLEGE

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

CS6702 GRAPH THEORY AND APPLICATIONS

QUESTION BANK

IV YEAR A & B / BATCH: 2015 -19

Vision of Institution: To build Jeppiaar Engineering College as an Institution of Academic Excellence in Technical education and Management education and to become a World Class University.

Mission of Institution

M1	To excel in teaching and **learning, research and innovation** by promoting the principles of scientific analysis and creative thinking
M2	To participate in the production, **development and dissemination of knowledge** and interact with **national and international communities**
M3	To equip students with values, ethics and life skills needed to enrich their lives and enable them to meaningfully contribute to the progress of society
M4	To prepare students for higher studies and lifelong learning, enrich them with the practical and entrepreneurial skills necessary to excel as future professionals and contribute to Nation's economy

Vision of Department: To emerge as a globally prominent department, developing ethical computer professionals, innovators and entrepreneurs with academic excellence through quality education and research.

Mission of Department

M1	To create **computer professionals** with an ability to identify and **formulate the engineering problems** and also to provide **innovative solutions** through **effective teaching learning process.**
M2	To **strengthen the core-competence** in computer science and engineering and to create an ability to **interact** effectively with industries.
M3	To produce engineers with good professional skills, **ethical values** and life skills for the **betterment of the society.**
M4	To encourage students towards **continuous and higher level learning** on technological advancements and provide a platform for **employment and self-employment.**

PROGRAM OUTCOMES (POs)

PO1	**Engineering Knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of computer science engineering problems.
PO2	**Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
PO3	**Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
PO4	**Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
PO5	**Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
PO6	**The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
PO7	**Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
PO8	**Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
PO9	**Individual and team work:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
PO10	**Communication:** Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
PO11	**Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
PO12	**Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PROGRAMME EDUCATIONAL OBJECTIVES (PEOs)

PEO 01: To address the real time complex engineering problems using innovative approach with strong core computing skills.

PEO 02: To apply core-analytical knowledge and appropriate techniques and provide solutions to real time challenges of national and global society.

PEO 03: Apply ethical knowledge for professional excellence and leadership for the betterment of the society**.**

PEO 04: Develop life-long learning skills needed for better employment and entrepreneurship.

PROGRAMME SPECIFIC OUTCOME (PSOs)

PSO1 – An ability to understand the core concepts of computer science and engineering and to enrich problem solving skills to analyze, design and implement software and hardware based systems of varying complexity.

PSO2 - To interpret real-time problems with analytical skills and to arrive at cost effective and optimal solution using advanced tools and techniques.

PSO3 - An understanding of social awareness and professional ethics with practical proficiency in the broad area of programming concepts by lifelong learning to inculcate employment and entrepreneurship skills.

BLOOM TAXANOMY LEVEL (BTL)

BTL6: Creating
BTL 5: Evaluating
BTL 4: Analyzing
BTL 3: Applying
BTL 2: Understanding
BTL 1: Remembering

CS6702 – Graph Theory and Applications

UNIT I

INTRODUCTION

Graphs – Introduction – Isomorphism – Sub graphs – Walks, Paths, Circuits – Connectedness – Components – Euler graphs – Hamiltonian paths and circuits – Trees – Properties of trees – Distance and centers in tree – Rooted and binary trees.

UNIT II

TREES, CONNECTIVITY & PLANARITY

Spanning trees – Fundamental circuits – Spanning trees in a weighted graph – cut sets – Properties of cut set – All cut sets – Fundamental circuits and cut sets – Connectivity and separability – Network flows – 1-Isomorphism – 2-Isomorphism – Combinational and geometric graphs – Planer graphs – Different representation of a planer graph.

UNIT III

MATRICES, COLOURING AND DIRECTED GRAPH

Chromatic number – Chromatic partitioning – Chromatic polynomial – Matching – Covering – Four color problem – Directed graphs – Types of directed graphs – Digraphs and binary relations – Directed paths and connectedness – Euler graphs

UNIT IV

PERMUTATIONS & COMBINATIONS :

Fundamental principles of counting - Permutations and combinations - Binomial theorem - combinations with repetition - Combinatorial numbers - Principle of inclusion and exclusion - Derangements - Arrangements with forbidden Positions.

UNIT V

GENERATING FUNCTIONS:

Generating functions - Partitions of integers - Exponential generating function – Summation operator - Recurrence relations - First order and second order – Non-homogeneous recurrence relations - Method of generating functions.

TEXT BOOKS:

1. Narsingh Deo, "Graph Theory: With Application to Engineering and Computer Science", Prentice Hall of India, 2003.
2. Grimaldi R.P. "Discrete and Combinatorial Mathematics: An Applied Introduction", Addison Wesley, 1994.

REFERENCES:

1. Clark J. and Holton D.A, "A First Look at Graph Theory", Allied Publishers, 1995.
2. Mott J.L., Kandel A. and Baker T.P. "Discrete Mathematics for Computer Scientists and Mathematicians" , Prentice Hall of India, 1996.
3. Liu C.L., "Elements of Discrete Mathematics", Mc Graw Hill, 1985.
4. Rosen K.H., "Discrete Mathematics and Its Applications", Mc Graw Hill, 2007.

COURSE OUTCOME

C402.1	**Recommend** precise and accurate mathematical definitions and proofs of graph.
C402.2	**Recommend** precise and accurate mathematical definitions and proofs of tree and graph applications.
C402.3	**Choose** combination of theoretical knowledge and independent mathematical thinking in creative investigation of questions in graph theory
C402.4	**Interpret** and **make use of** the concepts of counting Principles
C402.5	**Interpret** and **make use of** the concepts of recurrence relations and Generating Functions

INDEX

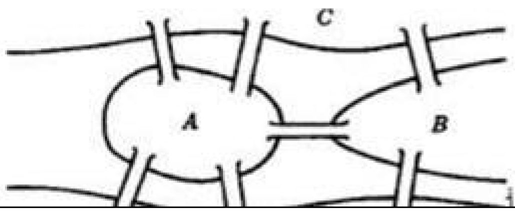
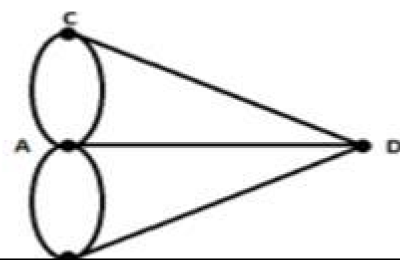
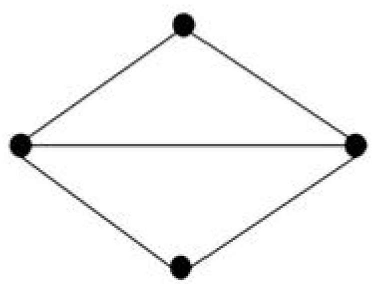
UNIT I

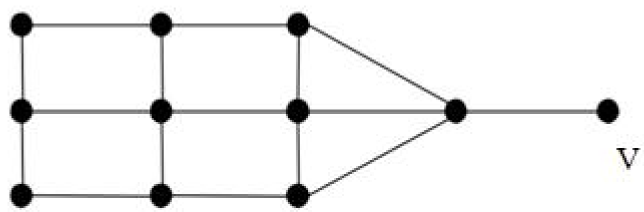
INTRODUCTION

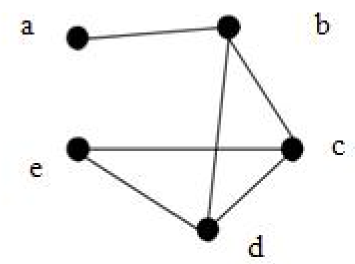
Graphs – Introduction – Isomorphism – Sub graphs – Walks, Paths, Circuits – Connectedness – Components – Euler graphs – Hamiltonian paths and circuits – Trees – Properties of trees – Distance and centers in tree – Rooted and binary trees.

S. No.	Question	Course Outcome	Blooms Taxonomy Level
1	**1.Define a Graph**. A graph is a ordered pair G=(V,E) where, V={v1,v2,v3...vn} is the vertex set whose elements are the vertices or nodes of the graph, denoted by V(G) or just V. E= ={e1,e2,e3...en} is the edge set whose elements are the edges or connections between vertices of the graph, denoted by E(G) or E.	C402.1	BTL 1
2	**2.List three situations where a graph could prove useful.** a) to represent air routes travelled among a certain set of cities by a particular airline. b) to represent an electrical network c) to represent a set of job applicants and a set of open positions in a corporation.	C402.1	BTL 1
3	**3.Define Walk with respect to graph.NOV/DEC 2016,APR/MAY2018** Let x, y be the vertices in an undirected graph G=(V,E). Walk : An x-y walk is a alternating sequence x=x0,e1,x1,e2......................en-1,xn-1,en,xn=y of vertices and edges from G, starting at vertex x and ending at vertex y and involving n edges ei={xi- 1,xi}, 1≤ i ≤ n. The length of a walk is its number of edges.	C402.1	BTL 1
4	**4.Define Path with respect to graph NOV/DEC 2016, APR/MAY2018** Path : If no vertex of the x-y walk occurs more than once, then the walk is called a x-y path.	C402.1	BTL 1

5	**5.Define Trail.** `Trail : If no edge in the x-y walk is repeated, then the walk is called a x-y trail.	C402.1	BTL 1
6	**Define Circuit. NOV/DEC 2016** Circuit : A closed x-x trail is called a circuit	C402.1	BTL 1
7	**Define degree(valency) of a vertex.** The number of edges incident on a vertex vi, with self loops counted twice is called the degree, deg(vi),of vertex vi.	C402.1	BTL 1
8	**When two edges and vertices are said to two be adjacent?** Two edges are said to be adjacent if they are incident on a common vertex. Two vertices are said to be adjacent if they are the end vertices of the same edge.	C402.1	BTL 1
9	**Define isolated vertex, pendant vertex and a null graph.** • Isolated Vertex-a vertex having no incident vertex is called as isolated vertex ,i.e vertices with degree zero. • Pendant Vertex-a vertex of degree one is called as pendant vertex. • Null graph-a graph with no edges is called as null graph.	C402.1	BTL 1
10	**When do we say that two graphs are isomorphic? Give an example.** Two graphs G and G' are said to be isomorphic (to each other) if there is one-to-one correspondence between their vertices and between their edges such that the incidence relationship is preserved.	C402.1	BTL 1
11	**What is a subgraph?** If G=(V,E) is a graph(directed or undirected), then G`1=(V1,E1) is called a subgraph of G if Φ≠V1 and E1, where each edge in E1 is incident with vertices in V1.	C402.1	BTL 1
12	**State the necessary conditions to show that two graphs are isomorphic.** The two graphs must have-same number of vertices a) same number of edges b) an equal number of vertices with a given degree	C402.1	BTL 1

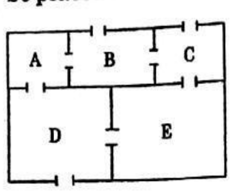
13	**Is it possible to have a 4-regular graph with 10 edges?** We have 4|V| = 20 v = 5 Thus, we have five vertices of degree 4	C402.1	BTL 1
14	**What is Konigsberg bridge problem?** The city of Königsberg in Prussia was set on both sides of the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. The problem was to devise a walk through the city that would cross each bridge once and only once.  	C402.1	BTL 1
15	**What are Euler trail and Euler circuit?** Let G=(V,E) be an undirected graph or multigraph with no isolated vertices. Then G is said to have an Euler circuit if there is a circuit in G that traverses every edge of the graph exactly once. If there is an open trail from a to b in G and this trail traverses each edge in G exactly once, the trail is called as Euler trail. 	C402.1	BTL 1
16	**What are the necessary and sufficient conditions to determine whether a given graph has an Euler circuit and Euler trail?** a) a given graph G will contain an Euler circuit if and only if all the vertices of G are of even degree. b) a given graph G will contain an Euler trail if and only if it contains atmost two vertices of odd degree.	C402.1	BTL 1

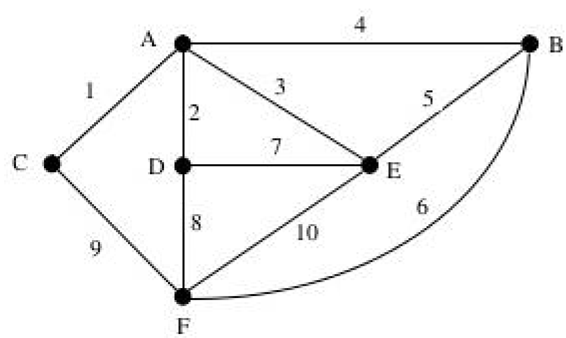
17	**What is an Hamilton circuit?** If G=(V,E) is a graph or multigraph with |V|≥3,we say that G has a Hamiltonian circuit if there is a circuit in G that contains every vertex in V i.e a closed walk that traverses every vertex of G exactly once, except the starting vertex, at which the walk terminates. The figure shows an example. A given graph may have more than one Hamiltonian circuit	C402.1	BTL 1
18	**What is a clique? State the properties of its vertices and edges.** A simple graph in which there exists an edge between every pair of vertices is called as clique. It is also referred as complete graph or Universal graph. The degree of every vertex in a complete graph G of n vertices is (n-1). The total number of edges in G is n(n-1)/2.	C402.1	BTL 1
19	**Does a Hamilton circuit exist on the graph below?**  Looking at the graph, if we start at vertex V, there is no way to visit each vertex exactly once and return back to V because there is only a single edge to V.	C402.1	BTL 1
20	**Prove that the sum of the degrees of the vertices of any finite graph is even.** Each edge ends at two vertices. If we begin with just the vertices and no edges, every vertex has degree zero, so the sum of those degrees is zero, an even number. Now add edges one at a time, each of which connects one vertex to another, or connects a vertex to itself. Either the degree of two vertices is increased by one (for a total of two) or one vertex's degree is increased by two. In either case, the sum of the degrees is increased by two, so the sum remains even.	C402.1	BTL 1
21	**Prove that a complete graph with n vertices contains n(n−1)/2 edges.** This is easy to prove by induction. If n = 1, zero edges are required, and 1(1−0)/2 = 0. Assume that a complete graph with k vertices has k(k −1)/2. When we add the (k + 1)st vertex, we need to connect it to the k original vertices, requiring k additional edges. We will then have -k(k−1)/2 + k = (k + 1)((k + 1)−1)/2 vertices. Hence the	C402.1	BTL 1

	proof.		
22	**Find the eccentricity of all the vertices in the given graph.**  E(a)=E(e)=3; E(b)=E(c)=E(d)=2	C402.1	BTL 1
23	**What is path length of a tree?** The path length of a tree is the sum of the path lengths from the root to all the pendant vertices.	C402.1	BTL 1
24	**Define Binary tree.** A binary tree is defined as a tree in which there is exactly one vertex of degree two, and each of the remaining vertices is of degree one or three	C402.1	BTL 1
25	**State the properties of binary tree. APR/MAY2018** . The two properties of the binary tree are- a) The number of vertices n in a binary tree is always odd. This is because there is exactly one vertex of even degree, and the remaining n-1 vertices are of odd degrees. b) Let p be the number of pendant vertices in a binary tree T. Then n-p-1 is the number of vertices of degree three. Therefore, the number of edges in T equals n-1 and hence p=(n+1)/2.	C402.1	BTL 1
26	**Define pendant vertex** • Pendant Vertex-a vertex of degree one is called as pendant vertex.	C402.1	BTL 1
27	**Define null graph.** • Null graph-a graph with no edges is called as null graph.	C402.1	BTL 1

28	**What is loop?** A loop is an edge whose vertices are equal	C402.1	BTL 1
29	**What is parallel edges?** The edges having the same pair of vertices.	C402.1	BTL 1
30	**What is meant by adjacent edges?** If two distinct edges are incident with a common vertex then they are called adjacent edges.	C402.1	BTL 1
31	**What is meant by eccentricity? NOV/DEC 2016** The eccentricity $\epsilon(v)$ of a graph vertex v in a connected graph G is the maximum graph distance between v and any other vertex u of G. For a disconnected graph, all vertices are defined to have infinite eccentricity	C402.1	BTL 4
32	**NOV/DEC 2017** **Determine the number of vertices for a graph G, which has 15 edges and each vertex has degree 6. Is the graph G be a simple graph ?**	C402.1	BTL 2
33	**NOV/DEC 2017** **Suppose G is a finite cycle-free connected graph with at least one edge. Show that G has at least two vertices of degree 1.**	C402.1	BTL 5
34	**1. Define Euler graph. Show that an Euler graph is connected except for any isolated vertices the graph may have.** A graph which contains an Eulerian circuit is called an Euler (Eulerian) graph. i.e., A circuit visits every edge exactly once which starts and ends on the same vertex. Since isolated vertices are present in Euler graph and every vertices must be even. i.e., every vertex is connected in an Euler graph. So an Euler graph is connected except for any isolated vertices. APR/MAY2017	C402.1	BTL 5

35	**2. Can there be a path longer than a Hamiltonian path (if any) in a simple, connected, undirected graph? Why?** No. There is no path longer than a Hamiltonian path. Hamiltonian path in a simple, connected, undirected graph is the longest path as it includes all the vertices. APR/MAY2017	C402.1	BTL 5
	PART B		
1	**Define a tree. Illustrate with example. State and prove the properties of trees. NOV/DEC 2016** Refer pg no 39 in Graph Theory by Narsingh Deo	C402.1	BTL 1
2	**Verify Euler's theorem with an example.** Refer pg no 23 in Graph Theory by Narsingh Deo	C402.1	BTL 1
3	**Solve Instant Insanity**. Refer pg no 18 in Graph Theory by Narsingh Deo	C402.2	BTL 1
4	**Explain about graph and applications of graph.** Refer pg no 3 in Graph Theory by Narsingh Deo	C402.1	BTL 1
5	**Explain Euler Circuit in detail.** Refer pg no 23 in Graph Theory by Narsingh Deo	C402.1	BTL 1
6	. **Explain Hamiltonian path with theorem proof.** Refer pg no 30 in Graph Theory by Narsingh Deo	C402.1	BTL 1
7	**Prove the following theorems** a)if graph has exactly two vertices of odd degree, there must be a path joining these two vertices. **NOV/DEC 2016** b)A simple graph with n vertices and k components can have at most (n-k) (n-k+1)/2		

	edges. c)There is one and only one path between every pair of vertices in a tree T. d)A tree with n vertices has n-1 edges. **APR/MAY2018** Show that the maximum no of edges in a simple graph with n vertices is n(n-1)/2. **NOV/DEC 2016** Prove that any two simple connected graphs with n vertices, all of degree two, are isometric **NOV/DEC 2016** Show that the Hamiltonian path is a spanning tree **NOV/DEC 2016** Prove that in any tree, there are at least 2 pendant vertices**. NOV/DEC 2016** Refer pg no 6 ,22, 42in Graph Theory by Narsingh Deo	C402.1	BTL 1
8.	**i.Prove that the number of vertices of add degree in a graph if always even APR/MAY2018(6)** **ii.prove that a connected graph G is an Euler graph if and only if and only if it can be decomposed into circuits. APR/MAY2018(10)** **iii.State and prove Dirac's theorem. APR/MAY2018(10)**	C402.1	BTL 6
9	**11. a) i)** The Figure 1 represents a floor plan with the doors between the rooms and the outside indicated. The real estate agent would like to tour the house, starting and ending outside, by going through each door exactly once. What is the fewest number of doors that should be added, and where should they be placed in order to make this tour possible ? Give reasons for your answer.  Fig. 1 **(8)** **ii)** Define closed-walk, open-walk, path and circuit. Take a graph of your choice and give an example to each one. **(8)** (OR) **b) i)** Nine members of committee have their dinner in round table. If no member sits near to the same neighbour more than once, how many days can this arrangement possible ? Write all possible arrangements. **(8)** **ii)** State four properties of a tree graph and prove them. **(8)**	C402.1	BTL 6

	NOV/DEC 2017		
10	(a) Define the following terms: (i) Walk (ii) Euler path (iii) Hamiltonian path (iv) Subgraph (v) Circuit (vi) Complete graph (6) From the given graph draw the following : (vii) Walk of length 6 (viii) Is this an Euler graph? Give reasons (ix) Is there a Hamiltonian path for this graph? Give reasons (x) Find atleast two complete subgraphs (10)  	C402.1	BTL 6
11	(i) List any five properties of trees (6) (ii) Define eccentricity of a vertex V in a tree T and give an example tree and its eccentricity from the root. (10)	C402.1	BTL 6

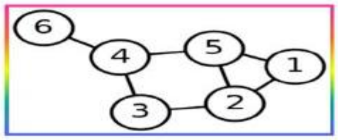
UNIT II

TREES, CONNECTIVITY & PLANARITY

Spanning trees – Fundamental circuits – Spanning trees in a weighted graph – cut sets – Properties of cut set – All cut sets – Fundamental circuits and cut sets – Connectivity and separability – Network flows – 1-Isomorphism – 2-Isomorphism – Combinational and geometric graphs – Planer graphs – Different representation of a planer graph.

PART-A

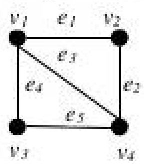
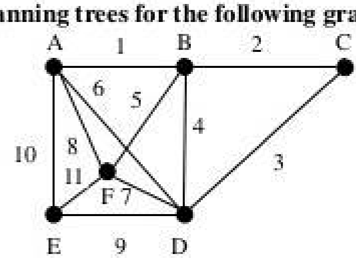
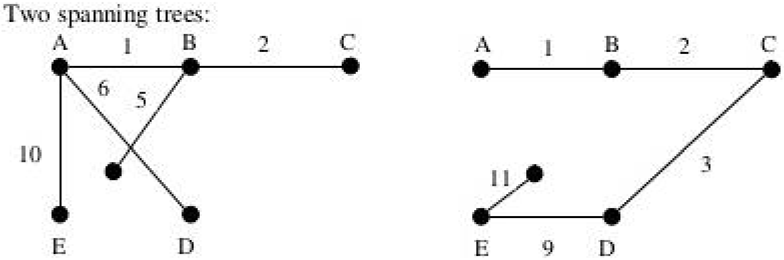
S. No.	Question	Course Outcome	Blooms Taxonomy Level
1	**Define Minimum Spanning Tree:** In a weighted graph, a minimum spanning tree is a spanning tree that has minimum weight that all other spanning trees of the same graph. In real world situations, this weight can be measured as distance, congestion, traffic load or any arbitrary value denoted to the edges.	C402.2	BTL 1
2	**Define Spanning Tree:** A spanning tree is a subset of Graph G, which has all the vertices covered with minimum possible number of edges. Hence, a spanning tree does not have cycles and it can not be disconnected.	C402.2	BTL 1
3	**Define the Term Rank .** Rank refers to the number of branches in any spanning tree G.r=n-k	C402.2	BTL 1
4	**Define the Term Nullity.** Nullity refers to the number of Chords in G.	C402.2	BTL 1
5	**What is a fundamental circuit?** A circuit formed by adding a chord to a spanning tree T, is called a fundamental circuit.	C402.2	BTL 1
6	**Define a cut set.** A cut set of a connected graph G is a set S of edges with the following properties: * The removal of all edges in S disconnects G. * The removal of some (but not all) of edges in S does not disconnects G.	C402.2	BTL 1

<table>
<tr><td>7</td><td>For the following graph given below draw the adjacency matrix

<table><tr><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th></tr><tr><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td></tr><tr><td>2</td><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr><tr><td>3</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td>4</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td></tr><tr><td>5</td><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>0</td></tr></table></td><td>C402.2</td><td>BTL 1</td></tr>
<tr><td>8</td><td>When a graph is said to be bipartite graph?
◦ If there exists a way to partition the set of vertices V, in the graph into two sets V1 and V2.where V1 □ V2 = V and V1 □ V2 = □, such that each edge in E contains one vertex from V1 and the other vertex from V2.</td><td>C402.2</td><td>BTL 1</td></tr>
<tr><td>9</td><td>What is a Vertex connectivity?
The connectivity (or vertex connectivity) K(G) of a connected graph G (other than a complete graph) is the minimum number of vertices whose removal disconnects G. When $K(G) \geq k$, the graph is said to be k-connected (or k-vertex connected). When a vertex is removed, all the edges incident to it are also removed.</td><td>C402.2</td><td>BTL 1</td></tr>
<tr><td>10</td><td>Define a bridge.
A bridge is a single edge whose removal disconnects a graph</td><td>C402.2</td><td>BTL 1</td></tr>
<tr><td>11</td><td>Define articulation point
A vertex in an undirected connected graph is an articulation point (or cut vertex) iff removing it (and edges through it) disconnects the graph. Articulation points represent vulnerabilities in a connected network – single points whose failure would split the network into 2 or more disconnected components. They are useful for designing reliable networks.</td><td>C402.2</td><td>BTL 1</td></tr>
<tr><td>12</td><td>What is a block?</td><td></td><td></td></tr>
</table>

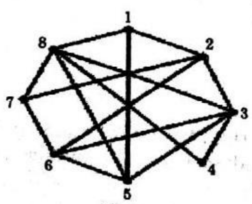
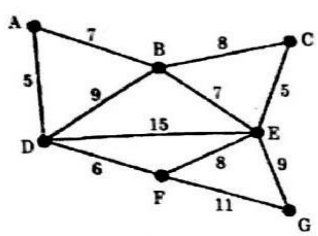
	A separable graph consists of two or more non separable sub graphs. Each of the largest non-separable sub graphs is called a block.	C402.2	BTL 1
13	**Define radius and diameter.** The eccentricity of a centre in a tree is defined as the radius of the tree.The diameter of the tree T, on the other hand is defined as the length of the longest path T.	C402.2	BTL 1
14	**Define Binary tree.** A binary tree is defined as the tree in which there is exactly one vertex of degree two, and each of the remaining vertices is of degree one or three.	C402.2	BTL 1
15	**Define Pendant Vertex**. A leaf vertex (also pendant vertex) is a vertex with degree one.	C402.2	BTL 1
16	**Define Kruskal's algorithm:** Start with no nodes or edges in the spanning tree, and repeatedly add the cheapest edge that does not create a cycle.	C402.2	BTL 1
17	**Define Prim's algorithm**: Start with any one node in the spanning tree, and repeatedly add the cheapest edge, and the node it leads to, for which the node is not already in the spanning tree	C402.2	BTL 1
18	**Define weighted graph.** A weighted graph is a graph for which each edge has an associated real number, called the weight of the edge. The sum of the weights of all of the edges is the total weight of the graph.	C402.2	BTL 1
19	**Define network flow.** A network flow graph G=(V,E) is a directed graph with two special vertices: the source vertex s, and the sink (destination) vertex t.	C402.2	BTL 1
20	**What is Branch?** An edge in a spanning tree T is called a Branch of T.	C402.2	BTL 1

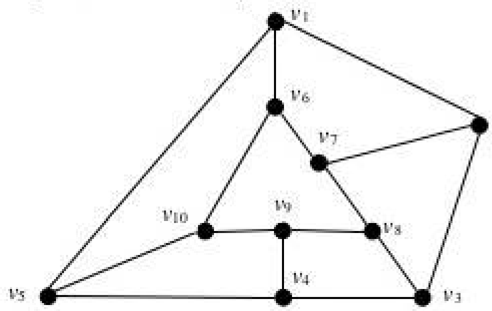
21	**What is Chord?** An edge of G that is not in a given spanning tree T is called a Chord.	C402.2	BTL 1
22	**What is elementary tree transformation?** The generation of one spanning tree to another through addition of a chord and deletion of an appropriate branch is called a cyclic interchange or element tree transformation	C402.2	BTL 1
23	**What is meant by fundamental cut-set?** A cut set S containing exactly one branch of a T tree is called fundamental cut-set with respect to T.	C402.2	BTL 1
24	**Write the properties of a cut set.** 1.Every cut set in a connected graph G must contain at least one branch of every spanning tree of G. 2.In a connected graph G,any minimal set of edges containing at least one branch of every spanning tree of G is a cut set.	C402.2	BTL 1
25	**Write the properties of a cut set.** 1.Every cut set in a connected graph G must contain at least one branch of every spanning tree of G. 2.In a connected graph G,any minimal set of edges containing at least one branch of every spanning tree of G is a cut set.	C402.2	BTL 1
26	**Define forest.** A collection of trees is called a forest.	C402.2	BTL 1
27	**What is meant by edge connectivity?** In a connected graph ,the number of edges in the smallest cut set is defined as the edge connectivity of G	C402.2	BTL 1
28	**What is meant by separability?** A connectivity graph is said to be separable if its vertex connectivity is one.	C402.2	BTL 1

29	**What is meant by non-separable graph?** A connectivity graph which is not separable is termed as non-separable graph.	C402.2	BTL 1
30	**Define 1-isomorphism. NOV/DEC 2016** Two graphs G1 and G2 are said to be 1-isomorphic if they become isomorphic to each other when we repeatedly decompose ,a cut-vertex into 2 vertices to produce 2 disjoint subgraphs.	C402.2	BTL 1
31	**Define 2-isomorphism. NOV/DEC 2016** Two graphs G1 and G2 are said to be 2-isomorphic if they become isomorphic after under going operation 1 or operation 2 or both any number of times.	C402.2	BTL 1
32	**What are the applications of planar graph? NOV/DEC 2016** We obtain a planar graph from a map by representing countries by vertices, and placing edges between countries that touch each other. Assuming each country is contiguous, this gives a planar graph.	C402.2	BTL 1
33	**Define fundamental circuit in a graph APR/MAY2018** A fundamental cut consists of E (T) \ e for a single edge in . That is, removing an edge from is a fundamental cut. The fundamental set of cuts consists of all such fundamental cuts for . Edit: There are vertices in K 3 , 3 , and so a spanning tree has edges.	C402.2	BTL 1
34	**State Kuratowski's theorem APR/MAY2018** Kuratowski's theorem states that a finite graph G is planar, if it is not possible to subdivide the edges of K5 or K3,3, and then possibly add additional edges and vertices, to form a graph isomorphic to G. Equivalently, a finite graph is planar if and only if it does not contain a subgraph that is homeomorphic to K5 ...	C402.2	BTL 1
35	**In a tree, every vertex is a cut-vertex. Justify the claim.** **NOV/DEC 2017** If is a **tree** of order at least , then contains a **cut vertex** such that **every vertex** adjacent to , with at most one exception is an end **vertex**. I know that if is a connected graph of order or more and it contains a bridge, then has a **cut vertex**.	C402.2	BTL 1

36	**A simple planar graph to which no edge can be added without destroying its planarity (while keeping the graph simple) is a maximal planar graph. Prove that every region in a maximal planar graph is a triangle.** **NOV/DEC 2017**	C402.2	BTL 1
	3. Define planar graphs. A planar graph is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints. Graph G: 	C402.2	BTL 1
	4. Identify two spanning trees for the following graph:  Two spanning trees: 	C402.2	BTL 1
	PART-B		
1	**i Write about Spanning trees with relevant theorems?** **ii Prove that every connected graph has atleast one spanning tree.NOV/DEC 2016**	C402.2	BTL 5

	Refer pg no 55 in Graph Theory by Narsingh Deo		
2	**Explain Spanning trees in a Weighted Graph.** Refer pg no 61 in Graph Theory by Narsingh Deo	C402.2	BTL 5
3	**Explain All Cut Sets in a Graph and give Theorem proof with an example.** Refer pg no 68 in Graph Theory by Narsingh Deo	C402.2	BTL 5
4	**Prove thatevery circuit has an even number of edges in a common with a cut-setsNOV/DEC 2016, APR/MAY2018(8)** Refer pg no 73 Graph Theory by Narsingh Deo	C402.2	BTL 5
5	**Explain Connectivity and Separability?** Refer pg no 75in Graph Theory by Narsingh Deo	C402.2	BTL 5
6	**Explain 2-Isomorphism and prove that the rank and nullity of a graph are invariant under 2- Isomorphism APR/MAY2018(8)** Refer pg no 80 in Graph Theory by Narsingh Deo	C402.2	BTL 5
7	**Explain 2-Isomorphism with examples?** Refer pg no 82 in Graph Theory by Narsingh Deo	C402.2	BTL 5
8	**a.Write Short notes on Network flows?** **b. Write about combinational and geometric Graphs?** Refer pg no 79 in Graph Theory by Narsingh Deo	C402.2	BTL 1
9	**iWrite in detail about different representation of a Planar graph?** **iiprove the graphs k5 and k3,3 are non planar.NOV/DEC 2016** Refer pg no 90in Graph Theory by Narsingh Deo	C402.2	BTL 1
10	**Evaluate using using prims and Kruskals algorithms** Refer pg no 6 1 Graph Theory by Narsingh Deo	C402.2	BTL 5

11	**Explain max-flow min-cut theorem? NOV/DEC 2016 ,APR/MAY2018(8)** Refer pg no 75 Graph Theory by Narsingh Deo	C402.2	BTL 5
12	**Prove that with respect to the given spanning tree T, a branch bi that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S and in no others.(8) APR/MAY2018**	C402.2	BTL 5
13	**12. a) i) Show that starting from any spanning tree of a graph G, every other spanning tree of G can be obtained by successive cyclic interchanges. (6)** **ii) Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets. (6)** **iii) Define edge vertex connectivity and edge connectivity. Give the relation between them. (4)** (OR) **b) i) Show, by drawing the graphs, that two graphs with the same rank and the same nullity need not be 2-isomorphic. (4)** **ii) State Kuratowski's Theorem and use it in order to prove the graph in Fig. 2 is non-planar. (8)**  **Fig. 2** **iii) State minimum cut maximum flow theorem. Using it calculate the maximum flow between the nodes D and E in the graph (Fig. 3). The number on a line represents the capacity. (4)**  **Fig. 3** **NOV/DEC 2017**	C402.2	BTL 5

14	i) Define spanning tree and give an example. (ii) A farm has six walled plots full of water. The graph representation of it is given below. Use the concepts of spanning tree, cut sets appropriately to determine the following: (1) How many walls will have to be broken so that all the water can be drained out? (2) If only one plot was full of water and this had to be drained into all other plots, then how many walls need to be broken? 	C402.2	BTL 5
15	State the Euler's formula relating the number of vertices, edges and faces of a planar connected graph. Give two conditions for testing for planarity of a given graph. Give a sample graph that is planar and another that is non-planar.	C402.2	BTL 5

UNIT III

MATRICES, COLOURING AND DIRECTED GRAPH

Chromatic number – Chromatic partitioning – Chromatic polynomial – Matching – Covering – Four color problem – Directed graphs – Types of directed graphs – Digraphs and binary relations – Directed paths and connectedness – Euler graphs

PART-A

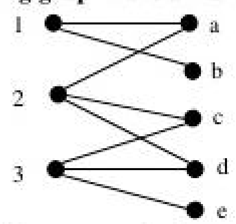
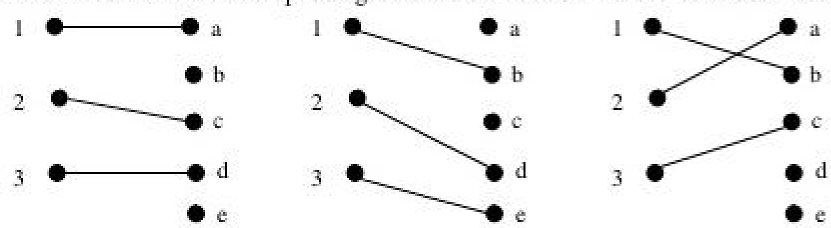
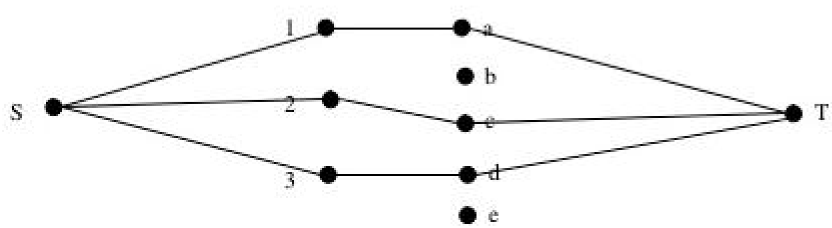
S. No.	Question	Course Outcome	Blooms Taxonomy Level
1	**Define Chromatic Number.** The chromatic number of a graph G is the smallest number of colors needed to color the vertices of so that no two adjacent vertices share the same color.	C402.3	BTL 1
2	**What is proper coloring?** Painting all the vertices of a graph with colors such that no two adjacent vertices have the same color is called proper coloring.	C402.3	BTL 1
3	**What is coloring problem?** Proper coloring of a graph with minimal number of color is called coloring problem	C402.3	BTL 1
4	**What is chromatic polynomial?** A properly colored graph G with n-vertices (many ways) and using large number of colors expressed by means of a polynomial, such a polynomial is called chromatic polynomial.	C402.3	BTL 1
5	**Write about maximal independent sets?NOV/DEC 2016** A maximal independent set is an set to which no other vertex can be added with out destroying its independence property.	C402.3	BTL 1
6	**Define chromatic partitioning.** A proper coloring of a graph naturally induces a partitioning of the vertices into different subsets	C402.3	BTL 1
7	**What is four color conjecture?** Any map(planar graph) can be properly colored with four colors.	C402.3	BTL 1
8	**What is dominating sets?** A dominating set (or externally stable set) in a graph G is a set of vertices that dominates every vertex v in G in the following sense. Either v is included in the dominating set or is adjacent to one or more vertices included in the dominating set.	C402.3	BTL 1

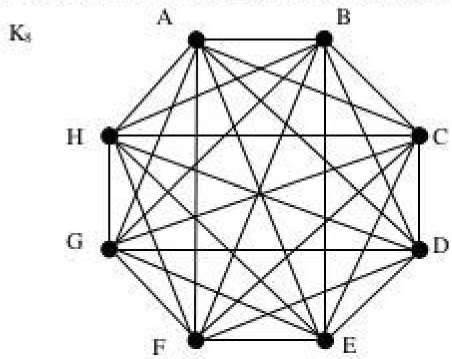
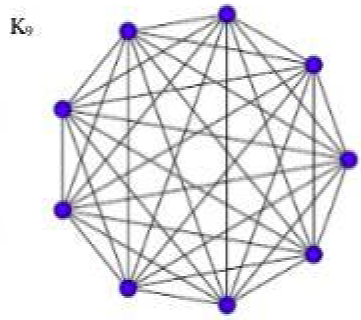
9	**Write about minimal dominating sets?NOV/DEC 2016** A minimal dominating set is a dominating set from which no vertex can be removed without destroying its dominance property.	C402.3	BTL 1
10	**Define Isomorphic digraphs:** Two digraphs are said to be isomorphic if their underlying graphs are isomorphic and the direction of the corresponding arcs are same	C402.3	BTL 1
11	**What is a maximal matching?** A maximal matching is a matching to which no edge in the graph can be added. Example: in a complete graph of three vertices (triangle) any single edge is a maximal matching.	C402.3	BTL 1
12	**What is an edge covering?** A set of edges that covers a graph G is said to be an edge covering, a covering sub graph or simply a covering of G.	C402.3	BTL 1
13	**What is Equivalence Relation.** A binary relation is said to be an equivalence relation if it is reflexive symmetric and transitive. Examples : “is parallel to”, “ is equal to” , “ is isomorphic to”.	C402.3	BTL 1
14	**Define digraph** A directed graph (or digraph) is a pair (V,E), where V is a non empty set and E is a set of ordered pairs of elements taken from the set V.	C402.3	BTL 1
15	**What is simple graph.** A digraph that has no self-loop or parallel edges is called a simple digraph.	C402.3	BTL 1
16	**Name the types of Simple Digraphs:** Asymmetric Digraphs: Symmetric Digraphs	C402.3	BTL 1

17	**What is meant by Asymmetric Digraphs** Asymmetric Digraphs: Digraphs that have at most one directed edge between a pair of vertices, but are allowed to have self-loops, are called asymmetric	C402.3	BTL 1
18	**What is meant by symmetric Digraphs** Symmetric Digraphs: Digraphs in which for every edge (a, b) (i.e., from vertex a to b) there is also an edge (b, a).	C402.3	BTL 1
19	**What is complete digraphs**. A complete undirected graph was defined as a simple graph in which every vertex is joined to every other vertex exactly by one edge.	C402.3	BTL 1
20	**Write the two types of complete graphs** For digraphs we have two types of complete graphs. A complete symmetric digraph is a simple digraph in which there is exactly one edge directed from every vertex to every other vertex, and a complete asymmetric digraph is an asymmetric digraph in which there is exactly one edge between every pair of vertices. A complete asymmetric digraph of n vertices contains n(n - 1)/2 edges, but a complete symmetric digraph of n vertices contains n(n - 1) edges. A complete asymmetric digraph is also called a tournament or a complete tournament.	C402.3	BTL 1
21	**Define a Euler Graph.** A closed walk in a graph G containing all the edges of G is called an Euler line in G. A graph containing an Euler line is called an Euler graph.	C402.3	BTL 1
22	**Prove:A connected graph G is an Euler graph if and only if all vertices of G are of even degree**. Proof : Necessity Let G(V, E) be an Euler graph. Thus G contains an Euler line Z, which is a closed walk. Let this walk start and end at the vertex u ∈ V. Since each visit of Z to an intermediate vertex v of Z contributes two to the degree of v and since Z traverses each edge exactly once, d(v) is even for	C402.3	BTL 1

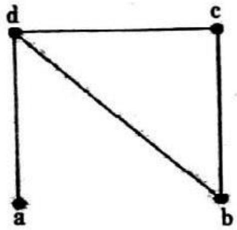
	every such vertex. Each intermediate visit to u contributes two to the degree of u, and also the initial and final edges of Z contribute one each to the degree of u. So the degree d(u) of u is also even.		
23	**Define Arborescence** A digraph G is said to be an arborescence if, i) G contains no circuit – neither directed nor semi circuit. ii) In G there is precisely one vertex v of zero – in degree.	C402.3	BTL 1
24	**What is strongly connected digraphs**. A digraph G is said to be strongly connected if there is at least one directed path from every vertex to every other vertex.	C402.3	BTL 1
25	**What is weakly connected digraphs**. A digraph G is said to be weakly connected if its corresponding undirected graph is connected but G is not strongly connected	C402.3	BTL 1
26	**What is meant by k-chromatic graph?** A graph that requires k different colors for its proper coloring and no less is called k-chromatic graph.	C402.3	BTL 1
27	**When a graph is said to be 2-chromatic?** A graph is 2-chromatic if and only if it is bipartite.	C402.3	BTL 1
28	**What is independent set?** A set of vertices in a graph is said to be an independent set of vertices of no two vertices in the set are adjacent.	C402.3	BTL 1
29	**What is independent number?** The number of vertices in the largest independent set of a graph G is called the independent number.	C402.3	BTL 1

30	**Define chromatic polynomial .** A polynomial which gives the number of different ways the graph G can be properly colored using the minimum number of colors	C402.3	BTL 1
31	Find the chromatic number of a complete graph of n vertices? NOV/DEC 2016 **K (k-1)$^{n-1}$**	C402.3	BTL 1
32	Let graph G is 2 –chromatic,then prove that it is bipartitle. **APR/MAY2018**	C402.3	BTL 1
33	Define minimal covering. **APR/MAY2018** Definition. Formally, an edge cover of a graph G is a set of edges C such that each vertex in G is incident with at least one edge in C. The set C is said to cover the vertices of G. ... The edge covering number is the size of a minimum edge covering.	C402.3	BTL 1
34	**Prove that a graph of n vertices is a complete graph iff its chromatic polynomial is $Pn(\lambda) = \lambda(\lambda-1)(\lambda-2)....(\lambda-n+1)$.** **NOV/DEC 2017**	C402.3	BTL 2
35	**Define the two types of connectedness in digraphs. Give examples.** **NOV/DEC 2017** wo kinds of connectedness, *strong* and *weak*. Strong connectedness of a directed graph is defined as follows: **Definition (Strong Connectedness of a Directed Graph)** A directed graph $G = (\mathcal{V}, \mathcal{E})$ is *strongly connected* if there is a path in *G* between every pair of vertices in $\mathcal{V}$. For example, Figure ❑ shows the directed graph $G_9 = \{\mathcal{V}, \mathcal{E}\}$ given by $\mathcal{V} = \{a, b, c, d, e, f\}$ $\mathcal{E} = \{(a, b), (b, c), (b, e), (c, a), (d, e), (e, f), (f, d), \}$ Notice that the graph G_9 is *not* connected! E.g., there is no path from any of the	C402.3	BTL 4

	vertices in $\{d, e, f\}$ to any of the vertices in $\{a, b, c\}$. Nevertheless, the graph ``looks" connected in the sense that it is not made of up of separate parts in the way that the graph $G_8$ in Figure ❑ is.  This idea of ``looking" connected is what *weak connectedness* represents. To define weak connectedness we need to introduce first the notion of the undirected graph that underlies a directed graph: Consider a directed graph $G = (\mathcal{V}, \mathcal{E})$. The underlying undirected graph is the graph $\widehat{G} = (\mathcal{V}, \widehat{\mathcal{E}})$ where $\widehat{\mathcal{E}}$ represents the set of undirected edges that is obtained by removing the arrowheads from the directed edges in *G*: $\widehat{\mathcal{E}} = \{ \{v, w\} : (v, w) \in \mathcal{E} \vee (w, v) \in \mathcal{E} \}.$		
36	**5. Does the following graph have a maximal matching? Give reason.**  Yes, the graph has a maximal matching. Three maximal matching are given. By maximum network flow concept 3 edges can be connected from source to destination.  	C402.3	BTL 4

	Draw K_8 and K_9 and show that thickness of K_8 is 2 while thickness of K_9 is 3.   The thickness of a graph G is the minimum number of planar graphs in which the edges of G can be partitioned. That is, if there exists a collection of k planar graphs, all having the same set of vertices, such that the union of these planar graphs is G, then the thickness of G is at most k. A planar graph ($K_1 - K_4$) has thickness 1. The Graphs ($K_5 - K_8$) of thickness 2 are called bi-planar graphs. The complete graph K_8 is a bi-planar graph. But K_9 is not a bi-planar graph. i.e., K_9 is tri-planar graph. So the thickness of K_9 is 3.		
	PARTB		
1	**Explain Chromatic number with necessary proof.** **i.Prove that every tree with 2 or more vertices is 2-chromatic. NOV/DEC 2016** **ii.Prove $P_n(\lambda) = \lambda(\lambda-1)(\lambda-2)\ldots(\lambda-n+1)$. NOV/DEC 2016** Refer pg no 165 Graph Theory by Narsingh Deo iii. **Prove that G is a tree with n then prove that its –chromatic polynomial is.** **$P_n(\lambda) = \lambda(\lambda-1)^{n-1}$ APR/MAY2018(8)** **iv Define Chromatic number Prove that a graph with atleast one edge is 2-chromatic if and only if it has no circuits of odd length. APR/MAY2018(8)**	C402.3	BTL 5
2	**Explain Chromatic Partitioning in detail.** Refer pg no 169 Graph Theory by Narsingh Deo	C402.3	BTL 5
3	**Explain Chromatic Polynomial with Theorem proof.**		

	Refer pg no 174 Graph Theory by Narsingh Deo	C402.3	BTL 5
4	**Write in detail about Matching in a graph**? Refer pg no 177 Graph Theory by Narsingh Deo	C402.3	BTL 1
5	**Explain in detail about Coverings?** **i.Prove that a covering g of a graph is minimal if g contains no paths opf lengh three or more.NOV/DEC 2016** Refer pg no 182 Graph Theory by Narsingh Deo	C402.3	BTL 5
6	**Explain five color problem with theorem proof**. **APR/MAY2018(8)** Refer pg no 186Graph Theory by Narsingh Deo	C402.3	BTL 5
7	**Explain directed graphs and its types? NOV/DEC 2016** Refer pg no 194 Graph Theory by Narsingh Deo	C402.3	BTL 5
8	**Explain in detail about digraphswith examples. APR/MAY2018(8).** Refer pg no 198 Graph Theory by Narsingh Deo	C402.3	BTL 5
9	**Explain directed paths and Connectedness**. Refer pg no 201 Graph Theory by Narsingh Deo	C402.3	BTL 5
10	**Explain Euler digraphs with theorem proof. NOV/DEC 2016** Refer pg no 203 Graph Theory by Narsingh Deo	C402.3	BTL 5

11	**13. a) i) Obtain the chromatic polynomial of the graph G in Fig. 4 using the theorem.** **$Pn(\lambda)$ of $G = Pn(\lambda)$ of $G' + Pn(\lambda)$ of G''.** **(8)**  **Fig. 4** **ii) State and prove five-color theorem.** **(8)** (OR) **b) i) Define the following and give one example to each :** **Complete Matching** **Minimal Covering** **Balanced Digraph** **Strongly Connected Digraph** **Fragment in a digraph.** **(10)** **ii) Prove that a digraph G is an Euler digraph if and only if G is connected and is balanced. Draw an example Euler digraph of 6 vertices.** **(6)** **NOV/DEC 2017**	C402.3	BTL 5
12	Describe the steps to find adjacency matrix and incidence matrix for a directed graph with a simple example.	C402.3	BTL 5
13	Write a note on chromatic polynomials and their applications.	C402.3	BTL 5

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UNIT IV

PERMUTATIONS & COMBINATIONS :

Fundamental principles of counting - Permutations and combinations - Binomial theorem - combinations with repetition - Combinatorial numbers - Principle of inclusion and exclusion - Derangements - Arrangements with forbidden Positions.

PART-A

S. No.	Question	Course Outcome	Blooms Taxonomy Level
1	**Define Fundamental Counting Principle:** The principle for determining the number of ways two or more operations can be performed together. Example:How many ways can six different books be positioned on a book shelf? $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ Six different books can be positioned 720 ways on a book shelf.	C402.4	BTL 1
2	**Explain Addition rule or Sum rule.** If one task or operation can be performed in m ways, while a second ask can be performed in n ways and the two tasks cannot be done simultaneously, then either of the tasks can be done in m + n ways. Example: There are 3 lists of computer projects consisting of 23, 15 and 19 possible projects respectively. No project is found in more than one list. How many possible projects can a student choose? Solution: Total number of projects is 23+15+19 = 57. Since no project is found in more than one list, the number of ways a project can be chosen is 57 ways.	C402.4	BTL 1
3	**Explain Multiplication rule or Product rule.** Suppose a certain task or operation can be done in m ways and another task independent of the former can be done in n ways. Then both of them can be done in	C402.4	BTL 1

	mn ways. Example: There are two different collections of books consisting of 6 Mathematics books and 4 Computer Science books. In how many ways can a student select a Mathematics book and a Computer Science book? Solution: One Mathematics book can be chosen in 6 ways and one Computer Science book can be chosen in 4 ways. □ Total number of ways taking one Mathematics book and one Computer Science book = (6)(4) = 24 ways.		
4	**Define Permutation**. A permutation is an arrangement of a given collection of objects in a definite order taking some of the objects or all the objects.	C402.4	BTL 1
5	**How many different bit strings of length 7 are there?** Solution: Each of the 7 places can be filled with 0 or 1. □the number of bit strings of length 7 is 27 =128	C402.4	BTL 1
6	**Define Combination**. A Combination is a selection of objects from a given collection of objects taken some objects or all the objects at a time. The order of selection is immaterial.	C402.4	BTL 1

7	**Explain Inclusion – Exclusion Principle**.
	
	A third basic principle of counting is the inclusion-exclusion principle.
	
	If A and B are mutually exclusive, then
	
	n(A□B) = n(A) + n(B) or |A□B| = |A| + |B|.
	
	If A and B are any two sets, then
	
	n(A□B) = n(A) + n(B) – n(A□B).
		C402.4	BTL 1

	If A, B, C are any three sets, then n(A□B □C) = n(A) + n(B) + n(C) – n(A□B) – n(B□C) – n(C□A) + n(A□B □C). It has applications in number theory, onto functions, derangements.		
8	**Explain Pigeon Hole Principle.** If m pigeons are assigned to n pigeonholes (m>n), then at least one pigeonhole has two or more pigeons.	C402.4	BTL 1
9	**A salesman at a computer store would like to display 5 models of personal computers, 4 models of computer monitors and 3 models of keyboards. In how many ways can he arrange them in a row if the items of the same kind are together?** Solution: The total number of arrangements = 3! x 5! x 4! x 3! = 103,680 .	C402.4	BTL 1
10	**From a college if we select 367 students, how many students have the same birthday?** Solution: There are 367 students and the maximum number of days in a year is 366 (leap year) and so 366 possible birthdays are there. Treating the 367 students as pigeons and the 366 birthdays as pigeonholes, by Pigeonhole principle atleast two students will have the same birthday.	C402.4	BTL 1
11	**How many different 7 digit phone numbers are possible if the 1st digit cannot be a 0 or 1?** 8*10*10*10*10*10*10=8000000 ways **12.Find the number of distinguishable permutations of the letters " MISSISSIPPI"** 11 letters with I repeated 4 times, S repeated 4 times, P repeated 2 times $\frac{11!}{4!*4!*2!} = \frac{39,916,800}{24*24*2} = 34,650$	C402.4	BTL 1
12	**Define Rook Polynomial.** The rook polynomial RB(x) of a board B is the generating		

	function for the numbers of arrangements of non-attacking rooks. RB(x) = ∑∞k=0 rk (B)xk where rk is the number of ways to place k non-attacking rooks on the board. Despite the notation, this is a finite sum, since the board is finite so there is a maximum number of non-attacking rooks it can hold; indeed, there cannot be more rooks than the smaller of the number of rows and columns in the board	C402.4	BTL 1
13	**What is a Inclusion Map?** Given a subset B of a set A the injection f: B → A is defined by f(b)=b for all b belongs to B is called the inclusion map.	C402.4	BTL 1
14	**From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there on the committee. In how many ways can it be done?** We may have (3 men and 2 women) or (4 men and 1 woman) or (5 men only). Required number of ways = (7C3 x 6C2) + (7C4 x 6C1) + (7C5) = 756	C402.4	BTL 1
15	**List all the derangements of the number 1, 2, 3, 4 and 5 where there first three numbers are 1, 2 and 3 in some order.** When 1, 2 and 3 are in some order there are only two derangements. 23154 and 31254.	C402.4	BTL 1
16	**In how many ways can 8 papers in an examination be arranged so that the two Mathematics papers are not consecutive?** Since the two Mathematics papers are not to be consecutive, remove them for the time being. Arrange the remaining 6 papers. This can be done in P(6,6) = 6! ways = 720 ways. In each of these arrangements of the 6 papers there are 7 gaps. In these gaps, the 2 Mathematics papers can be arranged in P(7,2) = (7)(6) = 42 ways. the total number of ways arranging all the 8 papers = 720 x 42 = 30240 ways.	C402.4	BTL 1
17	**How many different license plates are possible if each plate contains a sequence of 3 English alphabets followed by 3 digits? (Repetition is allowed)** There ar 26 alphabets and 10 digits. there ar 26 choices for each alphabet and 10 choices for each digit. all 3 alphabets can be chosen in 26 x 26 x 26	C402.4	BTL 1

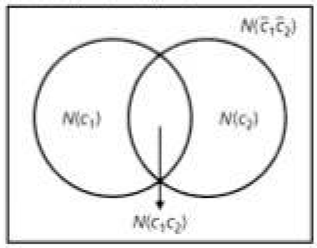
	= 263 ways,and the 3 numbers can be chosen in 10 x10 x 10 = 103 ways. □total number of possible license plates = 263 x 103 = 17,576,000		
18	**7 students enter a restaurant where each of them has one of the following:** a cheese burger, a chicken sandwich, a egg puff or a veg puff. How many different purchases are possible? (from restaurant point of view) n = 7, r = 4, no: of purchases = (n+r-1)Cr = (7+4-1)C4 =120.	C402.4	BTL 1
19	**Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?** Number of ways of selecting 3 consonants from 7= 7C3 Number of ways of selecting 2 vowels from 4 = 4C2 Number of ways of selecting 3 consonants from 7 and 2 vowels from 4 = 7C3 × 4C2 =(7×6×53×2×1)×(4×32×1)=210=(7×6×53×2×1)×(4×32×1)=210 It means we can have 210 groups where each group contains total 5 letters (3 consonants and 2 vowels).	C402.4	BTL 1
20	**Write the Binomial Theorem** $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$ $(1+x)^n = \overbrace{(1+x)}^{1}\overbrace{(1+x)}^{2}\cdots\overbrace{(1+x)}^{n}$ In order to get x^k, we have to choose x in k of {1, . . . , n}.	C402.4	BTL 1

	There are $\binom{n}{k}$ ways of doing this.		
21	**What is combinatorial number**. A combinatorial number is created by two positive integers' m and n written one on top of the other, within brackets: $\binom{m}{n}$	C402.4	BTL 1
22	**Write The formula that allows us to find the value of a combinatorial number.** $\binom{m}{n} = \frac{m!}{n! \cdot (m-n)!}$	C402.4	BTL 1
23	**Write some examples for combinatorial number.** $\binom{5}{2} = \frac{5!}{2! \cdot (5-2)!} = \frac{5 \cdot 4 \cancel{3 \cdot 2 \cdot 1}}{2 \cdot 1 \cancel{3 \cdot 2 \cdot 1}} = \frac{20}{2} = 10$ $\binom{4}{3} = \frac{4!}{3! \cdot (4-3)!} = \frac{4 \cancel{3 \cdot 2 \cdot 1}}{\cancel{3 \cdot 2 \cdot 1} \cdot 1} = 4$ $\binom{4}{4} = \frac{4!}{4! \cdot 0!} = 1$	C402.4	BTL 1
24	**What is meant by derangement?** A permutation of n distinct objects in which none of the objects is in its natural or original place is called derangement.	C402.4	BTL 1
25	**Find r in 6Pr= 360**	C402.4	BTL 1

	6Pr=6*5*4*3 r=4.		
26	**Find n if P(n,2)=72.** P(n,2)=72 n(n-1)=72 (n-9)(n+8)=0 n=9,n=-8 n=9.	C402.4	BTL 1
27	**Evaluate C(10,4).** C(10,4)=10!/(10-4) ! 4! =210	C402.4	BTL 1
28	**How many bytes contain exactly 2 i's.** 28	C402.4	BTL 1
29	**Find the number of derangements 1,2,3,4.** D4=4![1-1/1!+1/2!-1/3!+1/4!]=9.	C402.4	BTL 1
30	**A committee including 3 boys and 4 girls is to be formed from a group of 10 boys and 12 girls.How many different committees can be formed from the group? NOV/DEC 2016**	C402.4	BTL 1

	From 10 boys you have to select 3 boys, which could be done in 10C3 ways. Form 12 girls you've to select 4 girls, which could be done in 12C4 ways. Since you're to select 3 boys **and** 4 girls, you can do this in 10C3 x 12C4 ways!		
31	**In how ways can the letters of the word LEADING,TRIANGLE be arranged in such a that the vowels always come together.NOV/DEC 2016, APR/MAY2018** The **word** '**LEADING**' has 7 different **letters**. When the **vowels** EAI are **always together**, they **can** be supposed to form one **letter**. Then, we have to **arrange** the **letters** LNDG (EAI). ... The **vowels** (EAI) **can** be **arranged** among themselves in 3! = 6 **ways**.	C402.4	BTL 1
32	**Find the no of non negative integral solutions to X1+X2+X3+X4=20 (APR/MAY2018)** cases. case I : let x4=0 x1+x2+x3=20. You have to distribute 20 identical objects into 3 groups. No. of solutions to this = (20+3-1)C(3-1)=**22C2** case II : let x4=1 x1+x2+x3=16 You have to distribute 16 identical objects into 3 groups. No. of solutions to this = (16+3-1)C(3-1)=**18C2** case III : let x4=2 x1+x2+x3=12 You have to distribute 12 identical objects into 3 groups. No. of solutions to this = (12+3-1)C(3-1)=**14C2** case IV : let x4=3 x1+x2+x3=8 You have to distribute 8 identical objects into 3 groups. No. of solutions to this = (8+3-1)C(3-1)=**10C2** case V : let x4=4 x1+x2+x3=4 You have to distribute 4 identical objects into 3 groups. No. of solutions to this = (4+3-1)C(3-1)=**6C2** case VI : let x4=5 x1+x2+x3=0 You have to distribute 0 identical objects into 3 groups. No. of solutions to this = **2C2 =**	C402.4	BTL 1

	1 Hence total number of solutions = **1 +6C2 + 10C2 + 14C2 + 18C2 + 22C2** = 536		
33	**THALASSEMIA is a genetic blood disorder. How many ways can the letters in THALASSEMIA be arranged so that all three A's together ?** **NOV/DEC 2017**	C402.4	BTL 1
34	**Determine the number of positive integers n, $1 \le n \le 100$, that are not divisible by 3 or 7.** **NOV/DEC 2017**	C402.4	BTL 1
	7. State the rule of sum, the first principle of counting. **Rule of sum.** If the first task can be performed in m ways, while a second task can be performed in n ways, and the two tasks cannot be performed simultaneously, then performing either task can be accomplished in any one of $m + n$ ways. **Example** : A college library has 40 books on C++ and 50 books on Java. A student at this college can select 40+50=90 books to learn programming language. **Rule of Product** (counting principle) If a procedure can be broken into first and second stages, and if there are m possible outcomes for the first stage and if, for each of these outcomes, there are n possible outcomes for the second stage, then the total procedure can be carried out, in the designed order, in mn ways. **Example:** A drama club with six men and eight can select male and female role in 6 x 8 = 48 ways.	C402.4	BTL 1

	Use Venn diagram to represent the following scenario : **If S: a set, C_1= condition 1 and C_2 = condition 2 satisfied by some elements of S, indicate on the diagram - S, $N(C_1)$, $N(C_2)$, $N(C_1C_2)$ and $N(\overline{C_1}\,\overline{C_2})$.** 	C402.4	BTL 1
	PART B		
1	**Explain the fundamental principles of counting**. Refer page no 3 in Discrete mathematics by Girimaldi	C402.4	BTL 5
2	**A survey of 100 students with respect to their choice of the ice cream flavours Vanilla, Chocolate and Strawberry shows that 50 students like Vanilla, 43 like Chocolate, 28 like Strawberry, 13 like Vanilla and Chocolate, 11 like Chocolate and Strawberry, 12 like Strawberry and Vanilla, and 5 like all of them. Find the number of students who like** (i)Vanilla only (ii) Chocolate only (iii) Strawberry only (iv) Chocolate but not Strawberry(v) Chocolate and Strawberry but not Vanilla (vi) Vanilla or Chocolate, but not Strawberry. Also find the number of students who do not like any of these flavors. Refer notes	C402.4	BTL 2
3	**In making a seating arrangement for their son's wedding reception. Grace and Nick are down to four relatives, donated Ri for 1 ≤ i ≤ 4, who do not get along with one another. There is a single open seat at each of the five tables Tj where 1 ≤ j ≤ 5. Because of family differences,** a) R1 will not sit at T1 or T2. c) R2 will not sit at T2.	C402.4	BTL 1

	b) R3 will not sit at T3 or T4. d) R4 will not sit at T4 or T5. Refer notes		
4	**State and prove Binomial Theorem.** Refer page no 26 in Discrete mathematics by Girimaldi	C402.4	BTL 2
5	**Determine the number of positive intergers n, 1 ≤ n ≤ 2000, that are** i) Not divisible by 2,3 or 5. ii) Not divisible by 2,3,5 or 7. iii) Not divisible by 2, 3 or 5 but are divisible by 7. Refer notes	C402.4	BTL 1
6	**Explain the principles of Inclusion and exclusion.** Refer page no 209 in Discrete mathematics by Girimaldi	C402.4	BTL 5

7	**Consider the following program segment where i, j and k are integer variables**
	for i : = 1 to 20 do for j : = 1 to i do
	for k : = 1 to j do
	Print(i*j+k) How many times the print statement is executed in this program segment?
	Refer notes
		C402.4	BTL 1

8	**Explain Derangements.** Refer page no 134 in Discrete mathematics by Girimaldi	C402.4	BTL 5
9	**From a club consisting of 6 men 7 women, in how many ways can we select a committee of (i) 3men and 4 women(ii)4 persons which has at least 1 women (iii) 4 persons that has at most one man? (iv) 4 persons with people of both sexes?** Refer notes	C402.4	BTL 1
10	**.i. How many arrangements are there of all the vowels adjacent in SOCIOLOGICAL?NOV/DEC 2016** **ii.Find the value of n for the following:2P(n,2)+50=P(2n,2) NOV/DEC 2016** **iii.How many distinct four digit integers can one can make from the digits**	C402.4	BTL 1

	1,3,3,7,7and 8? NOV/DEC 2016 **iv.In how many possible ways could a student answer a 10 question true-false test. NOV/DEC 2016** REFER NOTES		
11	**i .How many arrangements of the letters in MISSISSIPPI has no consecutive S's? NOV/DEC 2016** **ii.A gym coach must select 11 seniors to play on a football team.If he can make his selection in 12,376 ways,how many seniors are eligible to play? NOV/DEC 2016** **iii How many permutations of size 3 can one produce with the letters m,r,a,f and t? NOV/DEC 2016** **iv.Rama has two dozen each of n colored beads.If she can select 20 beads (with repetitions of colors allowed) in 230,230 ways ,What is the value of n? NOV/DEC 2016** REFER NOTES	C402.4	BTL 1
12	**i.Using the principle of inclution and exclution find the number of prime numbers not exceeding 100 APR/MAY2018(8)** **ii.show that if n and k are positive integers,then** $C(n+1,k)=\frac{n+1}{k}C(n,k-1)$. **use this identity,construct an inductive definition of the binomial co-efficient APR/MAY2018(8)**	C402.4	BTL 1
13	**13.A survey of 150 college students reveals that 83 own cars 97 own bikes,28 own motorcycles,53 own a car and bike, 14 own a car anr motorcycle,7 own a bike and a motorcycle and 2 own all the three. How many students own a bike and nothing else and how many students do not own any of the three?** **APR/MAY2018(8)**	C402.4	BTL 1
14	**Five professor P1,P2,P3,P4,P5 are to be made class advisor for five sections C1,C2,C3,C4,c5 one professor for each section.p1 and p2 do not wish to become the class advisors for C1 or C2,P3 and P4 for C4 and P5 for C3 or C4 or C5.in how many ways can the professors be assigned the work(without displaying any professor)? APR/MAY2018(8)**	C402.4	BTL 1

<table>
<tr><td>15</td><td>

a) i) There are five students in a group and their roll numbers are S1, S2, S3, S4, S5 and S6. They are given with five assignments numbered 1 to 6. Each has to solve one assignment. How many ways the assignments can be distributed such that a student is not getting an assignment number same as his roll number? **(6)**

ii) Find the value of sum if the given program segment is executed.

```
main ()
{
   int  inc = 0, sum = 0;
   int i, j, k;
   for (i=1; i≤10; i++)
     for (j=1; j≤i; j++)
        for (k=1; k≤j; k++)
        {
        inc = inc + 1;
        sum = sum + inc;
        }
}
```

(10)

(OR)

b) i) Determine the coefficient of x^9y^6 in the expansion of $(4y - x)^{15}$. **(4)**

ii) How many integer solutions are possible for $x_1 + x_2 + x_3 + x_4 + x_5 < 40$ where $x_i \geq -3, 1 \leq i \leq 5$. **(6)**

iii) In a survey of the chewing gum tastes of a group of baseball players, it was found that 22 liked juicy fruit, 25 liked spearmint, 39 like bubble gum, 9 like both spearmint and juicy fruit, 17 liked juicy fruit and bubble gum, 20 liked spearmint and bubble gum, 6 liked all three and 4 liked none of these. How many baseball players were surveyed? **(6)**

NOV/DEC 2017

</td><td>C402.4</td><td>BTL 1</td></tr>
<tr><td>16</td><td>In how many ways can the 26 letters of the alphabet be permuted so that the patterns car, dog, pun or byte occures? Use the principle of inclusion and exclusion for this.</td><td>C402.4</td><td>BTL 1</td></tr>
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17	When n balls numbered $1, 2, 3 \ldots n$ are taken in succession from a container, a rencontre occurs if m[th] ball withdrawn is numbered m, $1 <= m <= n$. Find the probability of getting (i) no rencontres (ii) exactly one rencontre (iii) Atleast one rencontre and (iv) r rencontries $1 <= r <= n$. Show intermediate steps.	C402.4	BTL 1

UNIT V

GENERATING FUNCTIONS:

Generating functions - Partitions of integers - Exponential generating function – Summation operator - Recurrence relations - First order and second order – Non-homogeneous recurrence relations - Method of generating functions.

S. No.	Question	Course Outcome	Blooms Taxonomy Level
1	**Define Generating Function**. **NOV/DEC 2016** A generating function is a formal power series whose coefficients give the sequence .	C402.5	BTL 1
2	**Define Recurrence Relation. NOV/DEC 2016** A mathematical relationship expressing as some combination of with . When formulated as an equation to be solved, recurrence relations are known as recurrence equations, or sometimes difference equations.	C402.5	BTL 1
3	**Define Recursive Sequence.** A recursive sequence, also known as a recurrence sequence, is a sequence of numbers indexed by an integer and generated by solving a recurrence equation. The terms of a recursive sequences can be denoted symbolically in a number of different notations, such as , , or f[ ], where is a symbol representing the sequence.	C402.5	BTL 1
4	**If ek represents the number of ways to make change for k rupees, using Rs. 1, Rs. 2, Rs. 5, Rs. 10 and Rs. 100 , find the generating function for ek.**	C402.5	BTL 1

	f(x) = (Rs 1 factor) (Rs 2 factor) (Rs 5 factor) (Rs 10 Factor) (Rs 100 factor) =(1+x+x2+…) (1+x2+x4+…) (1+x5+x15+…) (1+x10+x20+…) (1+ x100+x200+…) =(1/(1-x)) (1/(1-x2)) (1/(1-x5)) (1/(1-x10)) (1/(1-x100))		
5	**Steps for solving a linear homogeneous recurrence relation of degree 2:** Step #1. Get the characteristic equation of the given recurrence relation. Step #2. Solve the characteristic equation to get the two roots r1, r2. The general solution is an = □1r1n + □2r2n Step #3. Substitute the initial conditions into the general solution to find the constants □1 and □2.	C402.5	BTL 1
6	**Solve the recurrence relation Fn = 5Fn−1 − 6Fn−2 where F0 = 1 and F1 = 4** The characteristic equation of the recurrence relation is − x2 − 5x + 6 = 0, So, (x − 3) (x − 2) = 0 Hence, the roots are − x1 = 3 and x2 = 2 The roots are real and distinct. So, this is in the form of case 1 Hence, the solution is − Fn = ax1n + bx2n Here, Fn = a3n + b2n (As x1 = 3 and x2 = 2) Therefore, 1 = F0 = a30 + b20 = a+b 4 = F1 = a31 + b21 = 3a+2b Solving these two equations, we get a = 2 and b = −1 Hence, the final solution is − Fn = 2.3n + (−1) . 2n = 2.3n − 2n	C402.5	BTL 1
7	**List some applications of Generating Functions.** • For solving a variety of counting problems. For example, the number of ways to make change for a Rs. 100 note with the notes of denominations Rs.1, Rs.2, Rs.5, Rs.10, Rs.20 and Rs.50. • For solving recurrence relations. • For proving some of the combinatorial identities.	C402.5	BTL 1

	• For finding asymptotic formulae for terms of sequences.		
8	**What is the generating function of the infinite series; 1, 1, 1, 1..?** Here, ak = 1, for 0 ≤k≤ ∞. Hence, G(x)=1+x+x2+x3+........=1(1−x). 9.Find the coefficient of 17 x in the expansion of (1+x5+x7)20 The only way to form an 17 x term is to gather two 5 x and one 7 x . Since there are 20C2 =190 ways to choose two 5 x from the 20 multiplicands and 18 ways to choose one 7 x from the remaining 18 multiplicands, the answer is 190* 18= 3420.	C402.5	BTL 1
9	**Explain all the three Methods to Solving Recurrences?** Iteration: Start with the recurrence and keep applying the recurrence equation until we get a pattern. The result is a guess at the closed form. Substitution: Guess the solution; prove it using induction. The result here is a proven closed form. It's often difficult to come up the guess so, in practice, iteration and substitution are used hand-in-hand. Master Theorem: Plugging into a formula that gives an approximate bound on the solution. The result here is only a bound on the closed form. It is not an exact solution.	C402.5	BTL 1
10	**If we have a recurrence relation for a sequence, is it possible to express the sequence in a way that does not use recursion?** Sometimes. When we are able to do so, we find what is called the closed form of the recurrence. It is an algebraic formula or a definition that tells us how to find the nth term without needing to know any of the preceding terms. The process of finding the closed form is called solving a recurrence.	C402.5	BTL 1
11	**Find a recurrence relation for the sequence 8, 24/7, 72/49, 216/343,…** Here a0=8 and each term is multiplied by 3/7 so an=(3/7)an-1 for n>1.	C402.5	BTL 1
12	**A bank pays 6% (annual) intrest on savings, compounding the interest monthly. If Raj deposits Rs. 1000/- on the first day of may, how much will he deposit be a worth year later?** Interest is 6% annually , so monthly is 6%/12=0.5% ==0.005 Pn+1=Pn+0.005Pn.	C402.5	BTL 1
13	**Define Catalan number.** In combinatorial mathematics, the Catalan numbers form a sequence of natural numbers that occur in various counting problems, often involving recursively-defined objects.	C402.5	BTL 1

	Cn = 1/(n+1) ∑ni=0 (n i)2		
14	**Find a12 if an+12=5an2 for n□0 and a0=2.** Let bn=an2. Then bn+1=5bn for n□0 and b0=4	C402.5	BTL 1
15	**For the alphabet={0,1,2,3}, how many strings of length n contains an even number of 1's** **Consider the nth symbol of a string of length n** (1) The nth symbol is 0, 2, 3. 3an-1 (2) The nth symbol is 1. Then there must be an odd number of 1's among the first n-1 symbols. 4n-1-an-1 an=3an-1+(4n-1-an-1)=2 an-1+4n-1	C402.5	BTL 1
16	**Solve the recurrence relation an+2-4an-1+3an=-200 for n□0 and a0=3000 and a1=3300** The solution for an+2-4an-1+3an=0 is an(h)=c1(3n)+c2. (Let an(p)=An) The particular solution for an+2-4an-1+3an=-200 is an(p)=100n. The solution to the problem is an= c1(3n)+c2+100n Finally, we have an= 100(3n)+2900+100n	C402.5	BTL 1
17	**Solve an = an-1 +8an-2 - 12an-3, with initial conditions a0 = 2, a1 = 3, and a2 = 19.** Let $g(x) = \sum^{\infty}_{n=0} a_n x^n$ $= 2 + 3x + 19x^2 + \sum^{\infty}_{n=3} a_n x^n$ $= 2 + 3x + 19x^2 + \sum^{\infty}_{n=3} (a_{n-1} + 8a_{n-2} - 12a_{n-3})x^n$ $= 2 + 3x + 19x^2 + \sum^{\infty}_{n=3} a_{n-1} x^n + 8\sum^{\infty}_{n=3} a_{n-2} x^n - 12\sum^{\infty}_{n=3} a_{n-3} x^n$ $= 2 + 3x + 19x^2 + x\sum^{\infty}_{n=3} a_{n-1} x^{n-1} + 8x^2 \sum^{\infty}_{n=3} a_{n-2} x^{n-2} - 12x^3 \sum^{\infty}_{n=3} a_{n-3} x^{n-3}$ $= 2 + 3x + 19x^2 + x\sum^{\infty}_{k=2} a_k x^k + 8x^2 \sum^{\infty}_{k=1} a_k x^k - 12x^3 \sum^{\infty}_{k=0} a_k x^k$ $= 2 + 3x + 19x^2 + x(\sum^{\infty}_{k=0} a_k x^k) - x(a_0 + a_1 x) + 8x^2 (\sum^{\infty}_{k=0} a_k x^k) - 8x^2 (a_0) - 12x^3 (\sum^{\infty}_{k=0} a_k x^k)$ $= 2 + 3x + 19x^2 + xg(x) + 8x^2 g(x) - 12x^3 g(x) - 2x - 3x^2 - 16x^2$ $= 2 + x + xg(x) + 8x^2 g(x) - 12x^3 g(x)$ Therefore, g(x)-xg(x)-8x2g(x)+12x3g(x) = 2+x. Therefore, g(x)-xg(x)-8x2g(x)+12x3g(x) = 2+x.	C402.5	BTL 1

	That's, g(x) =(2+x)/(1-x-8x2+12x3).		
18	**Properties of generating functions.** If p_s is normalized, then $g(x=1)=1$. and $g(x) \leq 1$ for every $\|x\| \leq 1$. All p_s's can be obtained from $g(x)$ using derivatives $1\ d^s g(x)\|$	C402.5	BTL 1
19	**Define Exponential generating functions.** An exponential generating function for the integer sequence , , ... is a function such that $E(y)=\sum_{k=0}^{\infty} a_k \frac{x^k}{k!}$ $a_0 + a_1 \frac{x}{1!} + a_2 \frac{x^2}{2!} + \ldots.$	C402.5	BTL 1
20	Find the generating functions for the following sequence. (a) 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, … The generating function is: $G(x) = 1 + 1x + 1x^2 + 1x^3 + 1x^4 + 1x^5 + 0x^6 + 0x^7 + \cdots$ $= 1 + x + x^2 + x^3 + x^4 + x^5$ We can apply the formula for the sum of a geometric series to rewrite G(x) as G(x)= $\frac{1-x^6}{1-x}$	C402.5	BTL 1

21	**What is meant by partition of integers?** A partition of a positive integer n is a multi-set of positive integers that sum to n. We denote the number of partitions of n by p_n.	C402.5	BTL 1
22	**Write the partitions of 5 .** 5 4 + 1 3 + 3 3 + 1 + 1 2 + 2 + 1 2 + 1 + 1 + 1 1 + 1 + 1 + 1 + 1. Thus p_5=7.	C402.5	BTL 1
23	**Write the types of solving recurrence relation** 1.Substitution 2.Iteration 3.Master Theorem	C402.5	BTL 1
24	**Define recursive definition.** A **recursive definition** of a concept is a definition that has two parts: 1. **Base**: A definition of the concept for some initial point, usually an n= 0 or n = 1 term. 2.**Recursion**: How to get to the next instance of the concept based on previous instances or smaller choices of n.	C402.5	BTL 1
25	**What is Dot diagram or Ferrers graph.** A Ferrers diagram represents partitions as patterns of dots, with the nth row having the same number of dots as the nth term in the partition.	C402.5	BTL 1

26	**Define second-order linear homogeneous recurrence relation with constant coefficients** A **second-order linear homogeneous recurrence relation with constant coefficients** is a recurrence relation of the form $a_k = A \cdot \boldsymbol{a_{k-1}} + B \cdot \boldsymbol{a_{k-2}}$ for all integers $k \geq$ some fixed integer, where A and B are fixed real numbers with $B \neq 0$.	C402.5	BTL 1
27	**. Define constant-recursive sequence** A constant-recursive sequence is a sequence satisfying a recurrence of this form. There are *d* degrees of freedom for solutions to this recurrence, i.e., the initial values can be taken to be any values but then the recurrence determines the sequence uniquely.	C402.5	BTL 1
28	**How to solve the non-homogeneous linear recurrence relations with constant coefficients.** If the recurrence is non-homogeneous, a particular solution can be found by the method of undetermined coefficients and the solution is the sum of the solution of the homogeneous and the particular solutions. Another method to solve an non-homogeneous recurrence is the method of symbolic differentiation.	C402.5	BTL 1
29	**Write the recurrence satisfied by the Fibonacci numbers.** The recurrence satisfied by the Fibonacci numbers is the archetype of a homogeneous linear recurrence relation with constant coefficients (see below). The Fibonacci sequence is defined using the recurrence	C402.5	BTL 1
30	**Find the exponential generating function of the sequence 0!,1!,2!,3!....(APR/MAY2018)** For a fixed *n* and fixed numbers of the letters, we already know how to do this. For example, if we have 3 *a*s, 4 *b*s, and 2 *c*s, there are (9345) such permutations. Now consider the following function: ∑*i*=0∞*x*2*i*+1(2*i*+1)!∑*i*=0∞*x*2*i*(2*i*)!∑*i*=0∞*xi*i!. What is the coefficient of *x*9/9! in this product? One way to get an *x*9 term is *x*33!*x*44!*x*22!=9!3!4!2!*x*99!=(9345)*x*99!. That is, this one term counts the number of permutations in which there are 3 *a*s, 4 *b*s, and 2 *c*s. The ultimate coefficient of *x*9/9! will be the sum of many such terms, counting the contributions of all possible choices of an odd number of *a*s, an even	C402.5	BTL 1

	number of *b*s, and any number of *c* s. Now we notice that $\sum_{i=0}^{\infty}\frac{x^i}{i!}=e^x$, and that the other two sums are closely related to this. A little thought leads to $e^x+e^{-x}=\sum_{i=0}^{\infty}\frac{x^i}{i!}+\sum_{i=0}^{\infty}\frac{(-x)^i}{i!}=\sum_{i=0}^{\infty}\frac{x^i+(-x)^i}{i!}$. Now $x^i+(-x)^i$ is $2x^i$ when i is even, and 0 when x is odd. Thus $e^x+e^{-x}=\sum_{i=0}^{\infty}\frac{2x^{2i}}{(2i)!}$, so that $\sum_{i=0}^{\infty}\frac{x^{2i}}{(2i)!}=\frac{e^x+e^{-x}}{2}$. A similar manipulation shows that $\sum_{i=0}^{\infty}\frac{x^{2i+1}}{(2i+1)!}=\frac{e^x-e^{-x}}{2}$. Thus, the generating function we seek is $\frac{e^x-e^{-x}}{2}\frac{e^x+e^{-x}}{2}e^x=\frac{1}{4}(e^x-e^{-x})(e^x+e^{-x})e^x=\frac{1}{4}(e^{3x}-e^{-x})$.		
31	**Determine the coefficient of x^{15} inf(X)=($X^2+X^3+X^4+\dots)^4$(APR/MAY2018)** $(1+x+\cdots+x^5)^8=\left(\frac{1-x^6}{1-x}\right)^8=(1-x^6)^8(1-x)^{-8}$ Using the binomial theoerem, $(1-x^6)^8=\sum_{k\ge0}(-1)^k\binom{8}{k}x^{6k}$ and using the negative binomial theorem, $(1-x)^{-8}=\sum_{k\ge0}(-1)^k\binom{-8}{k}x^k=\sum_{k\ge0}\binom{8+k-1}{k}x^k$ Thus, when we convolve the above two generating functions, the x^{24} coefficient is $\binom{8}{0}\binom{8+24-1}{24}-\binom{8}{1}\binom{8+18-1}{18}+\binom{8}{2}\binom{8+12-1}{12}-\binom{8}{3}\binom{8+6-1}{6}+\binom{8}{4}\binom{8+0-1}{0}$ **Addendum:** If $a(x)=\sum_{n\ge0}a_nx^n$ and $b(x)=\sum_{n\ge0}b_nx^n$, then the x^{24} coefficient of $c(x)=a(x)b(x)$ is $\sum_{k=0}^{24}a_kb_{n-k}$ The final answer I wrote then comes from setting $a(x)=(1-x^6)^8$, $b(x)=(1-x)^{-8}$, and realizing that $a_k=0$ unless k is a multiple of 6, so the above can be rewritten $\sum_{\ell=0}^{4}a_{6\ell}b_{n-6\ell}$	C402.5	BTL 1
32	**Find the coefficient of x^6 in $(3-5x)^{-8}$** **NOV/DEC 2017** Given expression is $(x - x^2)^{10}$	C402.5	BTL 1

	$\therefore\ T_{r+1} = {}^{10}C_r x^{10-r}(-x^2)^r = (-1)^r {}^{10}C_r x^{10-r} x^{2r} = (-1)^r {}^{10}C_r x^{10+r}$ For the coefficient of x^{15}, we have $10 + r = 15 \Rightarrow r = 5$ $\therefore\ T_{5+1} = (-1)^5 {}^{10}C_5 x^{15}$ $\therefore$ Coefficient of $x^{15} = -\frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!} = -252$		
33	**The number of virus affected files in a system is 500 (approximately) and this doubles every four hours. Using a recurrence relation, determine the number of virus affected files in the system after one day.** **NOV/DEC 2017**	C402.5	BTL 1
34	**Give explanation for the following :Generating function for the no. of ways to have n cents in pennies and nickels = $(1 + x + x^2 + \cdots)(1 + x^5 + x^{10} + \cdots)$** For pennies, the sequence ⟨1, 1, 1, …⟩ corresponds to the formal power series and the generating function is $1 + x + x^2 + \ldots = \sum_{n\geq0} x^n = \frac{1}{1-x}$. For nickels, the sequence is ⟨1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, …⟩ corresponds to another geometric series and the generating function is $1 + x^5 + x^{10} + \ldots = \sum_{n\geq0} x^{5n} = \sum_{n\geq0} (x^5)^n = \frac{1}{1-x^5}$. For pennies and nickels, $(1 + x + x^2 + \ldots)(1 + x^5 + x^{10} + \ldots)$ Clearly, the generating function is $\frac{1}{(1-x)(1-x^5)}$.	C402.5	BTL 1
35	**Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n,\ n \geq 0, a_0 = 3$.** Let n=0, $a_1 - a_0 = 0 \Rightarrow a_1 = a_0 \Rightarrow a_1 = 3$ i.e., $3n^2 - n = 0 \Rightarrow 3n - 1 = 0 \Rightarrow n = 1/3$	C402.5	BTL 1

PART B			
1	**1.Explain generating Functions.** Refer page no 229 in Discrete mathematics by Girimaldi	C402.5	BTL 5
2	**Find the generating function of the sequence a(n,r) for r ≥ 0 where a(n,r) denote the number of ways to select r objects without repetition from n distinct objects.** Refer notes	C402.5	BTL 5
3	**Write an algorithm for merge sorting. Show the intermediate steps when the numbers.** **310, 285, 179, 652, 351, 423, 861, 254, 450, 520 are sorted using Merge Sort and Derive the worst case analysis of Merge Sort using suitable illustrations.** Refer notes	C402.5	BTL 1
4	**Explain Exponential Generating functions. NOV/DEC 2016** Refer page no 230 in Discrete mathematics by Girimaldi	C402.5	BTL 5
5	**Explain recurrence relations.** Refer page no 243 in Discrete mathematics by Girimaldi	C402.5	BTL 5
6	**Explain first order linear recurrence relation.** Refer page no 250 in Discrete mathematics by Girimaldi	C402.5	BTL 5
7	**Explain Second order linear homogenous recurrence relation with case Distinct Real roots, complex roots and Repeated roots.** Refer page no 253 in Discrete mathematics by Girimaldi	C402.5	BTL 5
8	**Explain Nonhomogeneous Recurrence relation.** Refer page no 256 in Discrete mathematics by Girimaldi	C402.5	BTL 5
9	**The population of Mumbai city is 6,000,000 at the end of the year 2000. The number of immigrants is 20,000 n at the end of year n. The population of the city increased at the rate of of 5% per year.Use a recurrence relation to determine the population of the city at the end of 2010. NOV/DEC 2016** Refer notes	C402.5	BTL 5
10	**Derive the formula for the sum of the cubes of first n natural numbers using recurrence relation**. Refer notes	C402.5	BTL 5
11	**i Write short notes on summation operator**. **NOV/DEC 2016** **ii. Find the unique solution of the recurrence relation $6a_n-7a_{n-1}=0$,n>=1,a_3=343 NOV/DEC 2016** Refer notes	C402.5	BTL 5

12	**i.Obtain the fractional de-composition and identify the sequence having the expressing $\frac{3-5z}{1-2z-3z^2}$ as a generating function.(8) APR/MAY2018(4)**	C402.5	BTL 5
13	**ii.Find the generating function of the sequence 7,8,9,10…….(4) APR/MAY2018(4)** **iii.Find the number of distinct summands of the integer 6.(4) APR/MAY2018 (4)**	C402.5	BTL 5
14	**i.Solve the recurrence relation $y_{n+2}-6Y_{n+1}+8Y_{n=3n+5}$ APR/MAY2018** **ii.If a_n denotes the sum of the first n positive integer,find a recurrence relation for a_n and then solve it. APR/MAY2018(8)**	C402.5	BTL 5
15	a) i) Two cases of soft drinks, 24 bottles of one type and 24 bottles of another, are distributed among five surveyors who are conducting taste tests. In how many ways can the 48 bottles be distributed so that each surveyor gets at least two bottles of each type ? And in how many ways can they be distributed so that each surveyor gets at least two bottles of one type and three of other type ? Use generating function. (12) ii) Find all partitions of integer 6 and find the number of partitions with distinct summands. (4) (OR) b) i) A person invests Rs. 50,000 at 6% interest compounded annually. 1) Find the amount at the end of 1^{st}, 2^{nd}, 3^{rd} year. 2) Write the general explicit formula. 3) How long will it take to double the investment ? Use recurrence relation. (10) ii) Derive an explicit formula for the Fibonacci sequence using recurrence relation. (6) **NOV/DEC 2017**	C402.5	BTL 5
16	If a_n is count of number of ways a sequence of 1s and 2s will sum to n, for $n >= \theta$. Eg. $a_3 = 3$ (i) 1, 1, 1 ; (ii) 1, 2, and (iii) 2, 1 sum up to 3. Find and solve a sequence relation for a_n.	C402.5	BTL 5
17	What are Ferrers diagrams? Describe how they are used to (i) represent integer partition (ii) Conjugate diagram or dual partitions (iii) self-conjugates (iv) representing bisections of two partition.	C402.5	BTL 5