

JEPPIAAR ENGINEERING COLLEGE

Jeppiaar Nagar, Rajiv Gandhi Salai – 600 119

DEPARTMENT OF MECHANICAL ENGINEERING

QUESTION BANK



III SEMESTER

MA8353 Transforms and Partial Differential Equations

Regulation – 2017

SUBJECT : MA8353- TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

SEMESTER / YEAR: III / II

UNIT I - PARTIAL DIFFERENTIAL EQUATIONS

Formation of partial differential equations – Singular integrals -- Solutions of standard types of first order partial differential equations - Lagrange’s linear equation -- Linear partial differential equations of second and higher order with constant coefficients of both homogeneous and non-homogeneous types.

PART- A

Q.No.	Question	Bloom’s Taxonomy Level	Domain
1.	Form a partial differential equation by eliminating the arbitrary constants ‘a’ and ‘b’ from $z = ax^2 + by^2$. Solution $p=2ax$, $q=2by$ $a=p/2x$, $b=q/2y$ therefore PDE is $2z=px+qy$.	BTL -6	Creating
2.	Eliminate the arbitrary function from $z = f(y/x)$ and form the partial differential equation MA6351 M/J 2014, N/D ‘14 Solution: $px+qy=0$	BTL -6	Creating
3.	Form the PDE from $(x - a)^2 + (y - b)^2 + z^2 = r^2$. Solution Differentiating the given equation w.r.t x & y, $z^2[p^2+q^2+1]=r^2$.	BTL -3	Applying
4.	Find the complete integral of $p+q=pq$. Solution $p=a$, $q=b$ therefore $z=ax + \frac{a}{a-1}y + c$.	BTL- 6	Creating
5.	Form the partial differential equation by eliminating the arbitrary constants a, b from the relation $\log(az - 1) = x + ay + b$. A/M’15 Solution: $\log(az - 1) = x + ay + b$ Diff. p.w.r.t x&y, $\frac{ap}{az-1} = 1 - eqn1$ & $\frac{aq}{az-1} = a - eqn2$ $\frac{Eqn1}{Eqn2} \Rightarrow q = ap$ Sub in $a(z - p) = 1 \Rightarrow q(z - p) = p$	BTL -6	Creating
6.	Form the PDE by eliminating the arbitrary constants a,b from the relation $z = ax^3 + by^3$. MA6351 MAY/JUNE 2014 Solution: Differentiate w.r.t x and y $p = 3ax^2$, $q = 3by^2$ therefore $3z = px+qy$.	BTL -6	Creating
7.	Form a p.d.e. by eliminating the arbitrary constants from $z = (2x^2+a)(3y-b)$. Solution: $p = 4x(3y-b)$, $q = 3(2x^2+a)$ $3y - b = p/4x$ $(2x^2+a) = q/3$. Therefore $12xz = pq$.	BTL -6	Creating
8.	Form the partial differential equation by eliminating arbitrary function ϕ from $\phi(x^2 + y^2, z-xy) = 0$ [MA6351 M/J 2016] Solution: $u = x^2+y^2$ and $v = z-xy$. Then $u_x = 2x$, $u_y = 2y$; $v_x = p - y$;	BTL -6	Creating

	$v_y = q-x. \begin{vmatrix} u_x & u_y \\ v_x & v_x \end{vmatrix} = 0 \Rightarrow 2xq - 2x^2 - 2yp + 2y^2 = 0$		
9.	Form the partial differential equation by eliminating arbitrary constants a and b from $(x-a)^2 + (y-b)^2 + z^2 = 1$ Solution: Differentiating the given equation w.r.t x & y, $z^2[p^2+q^2+1]=1$	BTL -6	Creating
10.	Solve $[D^3-8DD'^2-D^2D'+12D'^3]z = 0$ [MA6351 M/J 2017] Solution: The auxiliary equation is $m^3-m^2-8m+12=0$; $m = 2, 2, -3$ The solution is $z = f_1(y+x)+f_2(y+2x)+xf_3(y+2x)$. [MA6351 NOV/DEC 2014]	BTL -3	Applying
11.	Find the complete solution of $q = 2px$ MA6351 APRIL/MAY 2015 Solution Find the complete solution of $q = 2px$ Solution: Let $q = a$ then $p = a/2x$ $dz = pdx + qdy$ $2z = a \log x + 2ay + 2b$.	BTL -3	Applying

12.	Find the complete solution of $p+q=1$ [MA6351 NOV/DEC 2014] Solution Complete integral is $z = ax + F(a) y + c$ Put $p = a, q = 1-a$. Therefore $z = px + (1-a) y + c$	BTL -3	Applying
13.	Find the complete solution of $p^3 - q^3 = 0$ MA6351 M/J 2016 Solution Complete integral is $z = ax + F(a) y + c$ Put $p = a, q = a$. Therefore $z = px + q y + c$	BTL -3	Applying
14.	Solve $[D^3+DD'^2-D^2D'-D'^3]z = 0$ Solution The auxiliary equation is $m^3-m^2+m-1=0$ $m = 1, -i, i \Rightarrow$ The solution is $z = f_1(y+x)+f_2(y+ix)+f_3(y-ix)$.	BTL -3	Applying
15.	Solve $(D-1)(D-D'+1)z = 0$. Solution $z = e^x f_1(y) + e^{-x} f_2(y+x)$	BTL -3	Applying
16.	Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} = 0$. Solution: A.E: $D[D-D'+1] = 0$ $h=0, h=k-1$ $z = f_1(y) + e^{-x} f_2(y+x)$	BTL -3	Applying
17.	Solve $(D^4 - D'^4)z = 0$. [MA6351 MAY/JUNE 2014] Solution: A.E : $m^4-1=0, m=\pm 1, \pm i$. $Z=C.F=f_1(y+x)+f_2(y-x)+f_3(y+ix)+f_4(y-ix)$.	BTL -3	Applying
18.	Solve $(D^2 - DD'+D'-1)Z = 0$. Solution: The given equation can be written as $(D-1)(D-D'+1)Z = 0$ $z = e^x f_1(y) + e^{-x} f_2(y+x)$	BTL -3	Applying
19.	Solve $x dx + y dy = z$.	BTL -3	Applying

	<p>Solution The subsidiary equation is $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$</p> $\frac{dx}{x} = \frac{dy}{y} \Rightarrow \log x = \log y + \log u$ $u = \frac{x}{y} \text{ Similarly } v = \frac{x}{z}.$		
20.	<p>Form the p.d.e. by eliminating the arbitrary constants from $z = ax + by + ab$</p> <p>Solution: $z = ax + by + ab$ $p = a$ & $q = b$ The required equation $z = px + qy + pq$.</p>	BTL -3	Applying

PART – B			
1.(a)	Find the PDE of all planes which are at a constant distance 'k' units from the origin.	BTL -6	Creating
1. (b)	Find the singular integral of $z = px + qy + 1 + p^2 + q^2$	BTL -2	Understanding
2. (a)	Form the partial differential equation by eliminating arbitrary function Φ from $\Phi(x^2 + y^2 + z^2, ax + by + cz) = 0$	BTL -6	Creating
2.(b)	Find the singular integral of $z = px + qy + p^2 + pq + q^2$	BTL -2	Understanding
3. (a)	Form the partial differential equation by eliminating arbitrary functions f and g from $z = x f(x/y) + y g(x)$	BTL -6	Creating
3.(b)	Find the singular integral of $z = px + qy + \sqrt{1 + p^2 + q^2}$.	BTL -3	Applying
MA6351 M/J 2016			
4. (a)	Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y)$. [MA6351 NOV/DEC 2014]	BTL -3	Applying
4.(b)	Form the partial differential equation by eliminating arbitrary function f and g from the relation $z = x f(x + t) + g(x + t)$	BTL -6	Creating
5. (a)	Solve $(D^2 - 2DD')z = x^3 y + e^{2x-y}$. [MA6351 NOV/DEC 2014]	BTL -3	Applying
5.(b)	Solve $x(y-z)p + y(z-x)q = z(x-y)$. [MA6351 NOV/DEC 2014]	BTL -3	Applying
6. (a)	Find the singular integral of $px + qy + p^2 - q^2$. MA6351 NOV/DEC 2014	BTL -2	Understanding
6.(b)	Find the general solution of $z = px + qy + p^2 + pq + q^2$. M/J '17	BTL -3	Applying
7. (a)	Find the complete solution of $z^2 (p^2 + q^2 + 1) = 1$	BTL -4	Analyzing

7. (b)	Find the general solution of $(D^2 + 2DD' + D'^2)z = 2\cos y - x\sin y$	BTL -2	Understanding
8. (a)	Find the general solution of $(D^2 + D'^2)z = x^2 y^2$	BTL -2	Understanding
8.(b)	Find the complete solution of $p^2 + x^2 y^2 q^2 = x^2 z^2$ A/M 2015	BTL -2	Understanding
9. (a)	Solve $(D^2 - 3DD' + 2D'^2)z = (2 + 4x)e^{x+2y}$	BTL -3	Applying
9.(b)	Obtain the complete solution of $z = px + qy + p^2 - q^2$ MA6351 A/M 2015	BTL -2	Understanding
10.(a)	Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$	BTL -3	Applying
10.(b)	Solve $(D^2 - 3DD' + 2D'^2)z = \sin(x + 5y)$	BTL -3	Applying
11(a)	Solve the Lagrange's equation $(x + 2z)p + (2xz - y)q = x^2 + y$	BTL -3	Applying
11(b)	Solve $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{2x+4y}$	BTL -3	Applying
12(a)	Solve $(D^2 + DD' - 6D'^2)z = y \cos x$	BTL -3	Applying
12(b)	Solve the partial differential equation $(x^2 - yz)p + (y^2 - xz)q = z^2 - xy$ MA6351 APRIL/MAY 2015, MA6351 M/J 2016	BTL -3	Applying
13(a)	Solve $(D^2 - DD' - 2DD'^2)z = e^{5x+y} + \sin(4x - y)$.	BTL -3	Applying
13(b)	Solve $(2D^2 - DD' - D'^2 + 6D + 3D')z = xe^y$	BTL -3	Applying
14(a)	Solve $(D^2 - 2DD')z = x^3 y + e^{2x-y}$	BTL -3	Applying
14(b)	Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y)$	BTL -3	Applying
15(a)	Form the PDE by eliminating the arbitrary function from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. [MA6351 MAY/JUNE 2014]	BTL -6	Creating
15(b)	Solve the Lagrange's equation $(x+2z)p+(2xz-y)q = x^2+y$. [MA6351 MAY/JUNE 2014]	BTL -3	Applying
16(a)	Solve $x^2 p^2 + y^2 q^2 = z^2$. [MA6351 MAY/JUNE 2014]	BTL -3	Applying
16(b)	Solve $(D^2 + DD' - 6D'^2)z = y \cos x$. [MA6351 MAY/JUNE 2014]	BTL -3	Applying

UNIT II - FOURIER SERIES: Dirichlet's conditions – General Fourier series – Odd and even functions
 – Half range sine series – Half range cosine series – Complex form of Fourier series – Parseval's identity
 – Harmonic analysis.

PART –A

Q.No	Question	Bloom's Taxonomy Level	Domain
1.	State the Dirichlet's conditions for a function $f(x)$ to be expanded as a Fourier series. MA6351 MAY/JUNE 2014, A/M 2017 Solution: (i) $f(x)$ is periodic, single valued and finite. (ii) $f(x)$ has a finite number of discontinuities in any one period (iii) $f(x)$ has a finite number of maxima and minima. (iv) $f(x)$ and $f'(x)$ are piecewise continuous.	BTL -1	Remembering
2.	Find the value of a_0 in the Fourier series expansion of $f(x)=e^x$ in $(0,2\pi)$. [MA6351 MAY/JUNE 2014] Solution: $a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx = \frac{1}{\pi} \int_0^{2\pi} e^x dx = 0.$	BTL -1	Remembering
3.	If $(\pi - x)^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ in $0 < x < 2\pi$, then deduce that value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$. [MA6351 NOV/DEC 2014] Solution: Put $x=0, \sum_{n=1}^{\infty} \frac{1}{n^2} = 6.$	BTL -1	Remembering

4.	Does $f(x) = \tan x$ posses a Fourier expansion? Solution No since $\tan x$ has infinite number of infinite discontinuous and not satisfying Dirichlet's condition.	BTL -2	Understanding
5.	Determine the value of a_n in the Fourier series expansion of $f(x) = x^3$ in $(-\pi, \pi)$. Solution: $a_n = 0$ since $f(x)$ is an odd function	BTL -5	Evaluating
6.	Find the constant term in the expansion of $\cos^2 x$ as a Fourier series in the interval $(-\pi, \pi)$. Solution: $a_0 = 1$	BTL -2	Understanding
7.	If $f(x)$ is an odd function defined in $(-l, l)$. What are the values of a_0 and a_n ? Solution: $a_n = 0 = a_0$	BTL -2	Understanding
8.	If the function $f(x) = x$ in the interval $0 < x < 2$ then find the constant term of the Fourier series expansion of the function f . Solution: $a_0 = 4\pi$	BTL -2	Understanding

9.	<p>Expand $f(x) = 1$ as a half range sine series in the interval $(0, \pi)$. [MA6351 MAY/JUNE 2014]</p> <p>Solution: The sine series of $f(x)$ in $(0, \pi)$ is given by</p> $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$ <p>where $b_n = \frac{2}{\pi} \int_0^{\pi} \sin nx dx = -\frac{2}{n\pi} [\cos nx]_0^{\pi} = 0$ if n is even</p> $= \frac{4}{n\pi} \text{ if } n \text{ is odd}$ $f(x) = \sum_{n=\text{odd}} \frac{4}{n\pi} \sin nx = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{(2n-1)}$	BTL -4	Analyzing
10.	<p>Find the value of the Fourier Series for $f(x) = 0 \quad -c < x < 0$ $= 1 \quad 0 < x < c$ at $x = 0$ MA6351 M/J 2016</p> <p>Solution: $f(x)$ at $x=0$ is a discontinuous point in the middle.</p> $f(x) \text{ at } x = 0 = \frac{f(0^-) + f(0^+)}{2}$ $f(0^-) = \lim_{h \rightarrow 0} f(0 - h) = \lim_{h \rightarrow 0} 0 = 0$ $f(0^+) = \lim_{h \rightarrow 0} f(0 + h) = \lim_{h \rightarrow 0} 1 = 1$ $\therefore f(x) \text{ at } x = 0 \rightarrow (0 + 1) / 2 = 1 / 2 = 0.5$	BTL -3	Applying
11.	<p>What is meant by Harmonic Analysis? Solution: The process of finding Euler constant for a tabular function is known as Harmonic Analysis.</p>	BTL -4	Analyzing
12.	<p>Find the constant term in the Fourier series corresponding to $f(x) = \cos^2 x$ expressed in the interval $(-\pi, \pi)$.</p> <p>Solution: Given $f(x) = \cos^2 x = \frac{1 + \cos 2x}{2}$</p> $\text{W.K.T } f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ <p>To find $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos^2 x dx = \frac{2}{\pi} \int_0^{\pi} \frac{1 + \cos 2x}{2} dx = \frac{1}{\pi} \left[x + \frac{\sin 2x}{2} \right]_0^{\pi}$</p> $= \frac{1}{\pi} [(\pi + 0) - (0 + 0)] = 1.$	BTL -1	Remembering
13.	<p>Define Root Mean Square (or) R.M.S value of a function $f(x)$ over the interval (a, b).</p> <p>Solution: The root mean square value of $f(x)$ over the interval (a, b) is defined as</p>	BTL -3	Applying

	$\text{R.M.S.} = \sqrt{\frac{\int_a^b [f(x)]^2 dx}{b-a}}$		
14.	<p>Find the root mean square value of the function $f(x) = x$ in the interval $(0,l)$.</p> <p><u>Solution:</u> The sine series of $f(x)$ in (a,b) is given by</p> $\text{R.M.S.} = \sqrt{\frac{\int_a^b [f(x)]^2 dx}{b-a}} = \sqrt{\frac{\int_0^l [x]^2 dx}{l-0}} = \frac{l}{\sqrt{3}}$	BTL -1	Remembering
15.	<p>If $f(x) = 2x$ in the interval $(0,4)$, then find the value of a_2 in the Fourier series expansion.</p> <p><u>Solution:</u> $a_2 = \frac{2}{4} \int_0^4 2x \cos[\pi x] dx = 0.$</p>	BTL -5	Evaluating
16.	<p>To which value, the half range sine series corresponding to $f(x) = x^2$ expressed in the interval $(0,5)$ converges at $x = 5$?</p> <p><u>Solution:</u> $x = 2$ is a point of discontinuity in the extremum.</p> $\therefore [f(x)]_{x=5} = \frac{f(0) + f(5)}{2} = \frac{[0] + [25]}{2} = \frac{25}{2}$	BTL -6	Creating
17.	<p>If the Fourier Series corresponding to $f(x) = x$ in the interval $(0, 2\pi)$ is $\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ without finding the values of a_0, a_n, b_n find the value of $\frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$.</p> <p>[MA2211 APR/MAY 2011]</p> <p><u>Solution:</u> By Parseval's Theorem</p> $\frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) = \frac{1}{\pi} \int_0^{2\pi} [f(x)]^2 dx = \frac{1}{\pi} \int_0^{2\pi} x^2 dx = \frac{1}{\pi} \left[\frac{x^3}{3} \right]_0^{2\pi} = \frac{8}{3} \pi^2$	BTL -4	Analyzing
18.	<p>Obtain the first term of the Fourier series for the function $f(x) = x^2, -\pi < x < \pi$.</p> <p><u>Solution:</u> Given $f(x) = x^2$, is an even function in $-\pi < x < \pi$.</p> <p>Therefore,</p> $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} x^2 dx = \frac{2}{\pi} \left[\frac{x^3}{3} \right]_0^{\pi} = \frac{2}{3} \pi^2.$	BTL -1	Remembering
19.	<p>Find the co-efficient b_n of the Fourier series for the function $f(x) = x \sin x$ in $(-2, 2)$.</p> <p><u>Solution:</u> $x \sin x$ is an even function in $(-2,2)$. Therefore $b_n = 0$.</p>	BTL -6	Creating

<p>20.</p>	<p>Find the sum of the Fourier Series for</p> $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 & 1 < x < 2 \end{cases} \text{ at } x = 1.$ <p><u>Solution:</u> $f(x)$ at $x=1$ is a discontinuous point in the middle.</p> $f(x) \text{ at } x = 1 = \frac{f(1^-) + f(1^+)}{2}$ $f(1^-) = \lim_{h \rightarrow 0} f(1 - h) = \lim_{h \rightarrow 0} 1 - h = 1$ $f(1^+) = \lim_{h \rightarrow 0} f(1 + h) = \lim_{h \rightarrow 0} 2 = 2$ $\therefore f(x) \text{ at } x = 1 \rightarrow (1 + 2) / 2 = 3 / 2 = 1.5$	<p>BTL -3</p>	<p>Applying</p>
<p>PART – B</p>			

<p>1.(a)</p>	<p>Obtain the Fourier's series of the function</p> $f(x) = \begin{cases} x & \text{for } 0 < x < \pi \\ 2\pi - x & \text{for } \pi < x < 2\pi \end{cases}$ <p>Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$ MA6351A/M 2017</p>	<p>BTL -1</p>	<p>Remembering</p>														
<p>1.(b)</p>	<p>Find the Fourier's series of $f(x) = x$ in $-\pi < x < \pi$</p> <p>And deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$</p>	<p>BTL -1</p>	<p>Remembering</p>														
<p>2.(a)</p>	<p>Find the Fourier's series expansion of period $2l$ for $f(x) = (l-x)^2$ in the range $(0, 2l)$. Hence deduce that</p> $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$	<p>BTL -2</p>	<p>Understanding</p>														
<p>2.(b)</p>	<p>Find the Fourier series of periodicity 2π for $f(x) = x^2$ in $-\pi \leq x \leq \pi$. Hence deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{90}$.</p>	<p>BTL -2</p>	<p>Understanding</p>														
<p>3.(a)</p>	<p>Find the Fourier series upto second harmonic for the following data:</p> <table border="1" data-bbox="367 1656 966 1753" style="margin-left: auto; margin-right: auto;"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>f(x)</td> <td>9</td> <td>18</td> <td>24</td> <td>28</td> <td>26</td> <td>20</td> </tr> </table> <p style="text-align: center;">MA6351 APRIL/ MAY 2017</p>	X	0	1	2	3	4	5	f(x)	9	18	24	28	26	20	<p>BTL -1</p>	<p>Remembering</p>
X	0	1	2	3	4	5											
f(x)	9	18	24	28	26	20											

3.(b)	Find the Fourier series of $f(x) = 2x - x^2$ in the interval $0 < x < 2$	BTL -1	Remembering														
4.(a)	Obtain the half range cosine series of the function $f(x) = \begin{cases} x & \text{in } \left(0, \frac{l}{2}\right) \\ l-x & \left(\frac{l}{2}, l\right) \end{cases}.$	BTL -5	Evaluating														
4.(b)	Find the half range sine series of the function $f(x) = x(\pi - x)$ in the interval $(0, \pi)$.	BTL -3	Applying														
5.(a)	Determine the Fourier series for the function $f(x) = \sin x $ in $-\pi \leq x \leq \pi$. MA6351 APRIL/ MAY 2015	BTL -5	Evaluating														
5.(b)	Find the complex form of the Fourier series of $f(x) = e^{-ax}$ in $(-1,1)$ MA6351 APRIL/ MAY 2017	BTL -1	Remembering														
6.(a)	Find the Fourier series for $f(x) = x \sin x$ in $(-\pi, \pi)$.	BTL -2	Remembering														
6.(b)	Find the Fourier series expansion of $f(x) = x + x^2$ $-2 \leq x \leq 2$.	BTL -2	Remembering														
7.(a)	Find the Fourier series for $f(x) = \begin{cases} x & (0, \pi/2) \\ \pi - x & (\pi/2, 2\pi) \end{cases}$. MA6351 APRIL/ MAY 2015	BTL -4	Analyzing														
7.(b)	Find the Fourier series of $f(x) = x + x^2$ in $(-1, 1)$ with period 2l.	BTL -3	Applying														
8.(a)	Find the Fourier series as far as the second harmonic to represent the function $f(x)$ with period 6, given in the following table.	BTL -6	Creating														
	<table border="1"> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>f(x)</td> <td>9</td> <td>18</td> <td>24</td> <td>28</td> <td>26</td> <td>20</td> </tr> </table>			X	0	1	2	3	4	5	f(x)	9	18	24	28	26	20
	X			0	1	2	3	4	5								
f(x)	9	18	24	28	26	20											

8.(b)	Find the complex form of the Fourier series of $f(x)=e^{-x}$ in $-1 < x < 1$. MA6351 APRIL/ MAY 2015	BTL -2	Remembering																												
9.(a)	Find the half range cosine series for the function $f(x) = x(\pi - x)$ in $0 < x < \pi$. Deduce $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{90}$	BTL -2	Remembering																												
9.(b)	Obtain the Fourier series to represent the function $f(x) = x , -\pi < x < \pi$ and deduce $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ (M/J 2012)	BTL -3	Applying																												
10.(a)	Find the half range sine series of $f(x) = lx - x^2$ in $(0,1)$ (N/D 2013)	BTL -1	Remembering																												
10.(b)	Obtain the Fourier cosine series expansion of $f(x) = x$ in $0 < x < 4$. Hence deduce the value of $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots$	BTL -1	Remembering																												
11.(a)	By using Cosine series show that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \dots = \frac{\pi^4}{96}$ for $f(x) = x$ in $0 < x < \pi$	BTL -4	Analyzing																												
11.(b)	Find the Fourier cosine series up to third harmonic to represent the function given by the following data: MA6351 M/J 2016 <table border="1" data-bbox="391 1157 1078 1255"> <tbody> <tr> <td>X</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>Y</td> <td>4</td> <td>8</td> <td>15</td> <td>7</td> <td>6</td> <td>2</td> </tr> </tbody> </table>	X	0	1	2	3	4	5	Y	4	8	15	7	6	2	BTL -6	Creating														
X	0	1	2	3	4	5																									
Y	4	8	15	7	6	2																									
12.(a)	Show that the complex form of Fourier series for the function $f(x)=e^{ax}$ $(-\pi, \pi)$	BTL -1	Remembering																												
12.(b)	Find the complex form of the Fourier series of $f(x)=e^{-x}$ in $-1 < x < 1$. (N/D 2009)	BTL -4	Analyzing																												
13.	Calculate the first 3 harmonics of the Fourier of $f(x)$ from the following data <table border="1" data-bbox="245 1696 792 1896"> <tbody> <tr> <td>x</td> <td>0</td> <td>30</td> <td>60</td> <td>90</td> <td>120</td> <td>150</td> <td>180</td> <td>210</td> <td>240</td> <td>270</td> <td>300</td> <td>330</td> <td>360</td> </tr> <tr> <td>f(x)</td> <td>1.8</td> <td>1.1</td> <td>0.3</td> <td>0.16</td> <td>0.5</td> <td>1.3</td> <td>2.16</td> <td>1.25</td> <td>1.3</td> <td>1.52</td> <td>1.76</td> <td>2</td> <td>1.8</td> </tr> </tbody> </table>	x	0	30	60	90	120	150	180	210	240	270	300	330	360	f(x)	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2	1.8	BTL -6	Creating
x	0	30	60	90	120	150	180	210	240	270	300	330	360																		
f(x)	1.8	1.1	0.3	0.16	0.5	1.3	2.16	1.25	1.3	1.52	1.76	2	1.8																		

14.(a)	Find the complex form of the Fourier series of $f(x) = e^{-s}$ in $-1 < x < 1$.	BTL -4	Analyzing											
14.(b)	Find the Fourier series up to the second harmonic from the following table.	BTL -6	Creating											
	<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td>f(x)</td> <td>9</td> <td>18</td> <td>24</td> <td>28</td> <td>26</td> <td>20</td> </tr> </table>			x	0	1	2	3	4	5	f(x)	9	18	24
x	0	1	2	3	4	5								
f(x)	9	18	24	28	26	20								

UNIT - III APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Solution of one dimensional wave equation-One dimensional heat equation-Steady state solution of two dimensional heat equation-Fourier series solutions in Cartesian coordinates .

Textbook : Grewal. B.S., "Higher Engineering Mathematics", 42nd Edition, Khanna Publishers, Delhi, 2012.

PART - A

Q.No	Questions	BT Level	Competence	PO
1	<p>What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation.</p> <p><u>Solution:</u> The correct solution of one dimensional wave equation is of periodic in nature. But the solution of heat equation is not periodic in nature.</p> <p>(A.U.N/D 2017, N/D 2011,2012, M/J 2013)</p>	BTL-4	Analyzing	PO1
2	<p>In steady state conditions derive the solution of one dimensional heat flow equations. [Nov / Dec 2005]</p> <p><u>Solution:</u> one dimensional heat flow equation is</p> $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2} \dots\dots\dots(1)$ <p>When the steady state conditions exists, put $\frac{\partial u}{\partial t} = 0$</p> <p>Then (1) becomes, $\frac{\partial^2 u}{\partial x^2} = 0$.</p> <p>Solving, we get $u(x)=ax+b$. a and b are arbitrary constants.</p>	BTL-2	Understanding	PO1,P O2,PO 3

3	<p>What are the possible solution of one dimensional wave equation.</p> <p><u>Solution:</u> The possible solutions are (i) $y(x,t)=(A_1e^{px} + A_2e^{-px})(A_3e^{pat} + A_4e^{-pat})$ (ii)</p> <p>$y(x,t)=(B_1 \cos px + B_2 \sin px)(B_3 \cos pat + B_4 \sin pat)$ (iii)</p> <p>$y(x,t)=(C_1x + C_2)(C_3t + C_4)$. (A.U. M/J 2014)</p>	BTL-1	Remembering	PO1,P O2
4	<p>Classify the P.D.E $3u_{xx} + 4u_{yy} + 3u_y - 2u_x = 0$.</p> <p><u>Solution:</u> $B^2 - 4AC = 16 - 4(3)(0) = 16 > 0$. It is hyperbolic. (A.U M/J 2008)</p>	BTL-1	Remembering	PO1
5	<p>The ends A and B of a rod of length 10cm long have their temperatures kept at $20^\circ C$ and $70^\circ C$. Find the Steady state temperature distribution of the rod.</p> <p><u>Solution:</u> The initial temperature distribution is $u(x,0) = \frac{b-a}{l}x + a$. Here $a = 20^\circ C, b = 70^\circ C, l = 10cm$.</p> <p>$\therefore u(x,t) = \frac{70-20}{10}x + 20 = 5x + 20, 0 < x < 10$.</p> <p>(A.U M/J 2008, A.U N/D 2012)</p>	BTL-1	Remembering	PO1
6	<p>Classify the PDE</p> $\frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} - 12 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + 7u = x^2 + y^2.$ <p><u>Solution:</u> The given PDE is $u_{xx} + 4u_{xy} + 4u_{yy} - 12u_x + u_y + 7u = x^2 + y^2$. $A=1; B=4; C=4$.</p> <p>$B^2 - 4AC = 16 - 16 = 0$.</p> <p>\therefore The given PDE is parabolic. (A.U M/J 2009)</p>	BTL-3	Applying	PO1
7	<p>Write down the one dimensional heat equation.</p> <p><u>Solution:</u> The one dimensional heat equation is $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2} \frac{\partial u}{\partial t}$</p> <p>(A.U M/J 2010,N/D 2012)</p>	BTL-1	Remembering	PO1,P O2
8	<p>Write down the possible solutions of one dimensional heat flow equation.</p> <p><u>Solution:</u> The various possible solutions of one dimensional heat equation are</p> <p>(i) $u(x,t) = (Ae^{px} + Be^{-px})e^{\alpha^2 p^2 t}$</p> <p>(ii) $(A \cos px + B \sin px)e^{-\alpha^2 p^2 t}$ (iii) $u(x,t) = (Ax+B)$.</p>	BTL-1	Remembering	PO1,P O2

	(A.U M/J 2009,A.U N/D 2014)			
9	<p>Write the one dimensional wave equation with initial and boundary conditions in which the initial position of the string is $f(x)$ and the initial velocity imparted at each point is $g(x)$.</p> <p><u>Solution:</u> The wave equation is $\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2}$. The boundary conditions are</p> <p>(i) $y(0,t)=0, \forall t>0$ (ii) $y(l,t)=0, \forall t > 0$</p> <p>(iii) $\frac{\partial y}{\partial t}(x, 0) = g(x), 0 < x < l$. (iv) $y(x,0)=f(x), 0<x<l$.</p> <p>(A.U N/D 2007,M/J 2012,2016)</p>	BTL-3	Applying	PO1
10	<p>Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2} \frac{\partial u}{\partial t}$</p> <p><u>Solution:</u> Given $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\alpha^2} \frac{\partial u}{\partial t}$</p> $\alpha^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ <p>Here $A = \alpha^2; B = 0; C = 0$.</p> $\therefore B^2 - 4AC = 0 - 4(\alpha^2)(0) = 0.$ <p>(A.U M/J 2007,M/J2013 ,M/J 2016)</p>	BTL-1	Remembering	PO1
11	<p>State the two dimensional Laplace equation?</p> <p><u>Solution:</u> $U_{xx} + U_{yy} = 0$</p> <p>(A.U., N/D 2011,2012, M/J 2014)</p>	BTL-1	Remembering	PO1
12	<p>In an one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ what does the constant stands for ?</p> <p><u>Solution:</u> α^2 is called the diffusivity of the material of the body through which the heat flows. If ρ be the density, c the specific heat and k thermal conductivity of the material, we have the relation $k / \rho c = \alpha^2$. (A.U M/J 2013)</p>	BTL-1	Remembering	PO1
13	<p>What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation.</p> <p><u>Solution:</u></p> <p>Solution of the one dimensional wave equation is of periodic in nature. But Solution of the one dimensional heat equation is not of periodic in nature.</p>	BTL-1	Remembering	PO1,P O2,PO 5

	(A.U A/M 2016)			
14	<p>In the wave equation $\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2}$, What does α^2 stands for ?</p> <p><u>Solution</u> : $\alpha^2 = \frac{\text{Tension}}{\text{MassperUnitlength}}$</p> <p>(A.U M/J 2014)</p>	BTL-1	Remembering	PO1
15	<p>In 2D heat equation or Laplace equation ,What is the basic assumption?</p> <p><u>Solution</u> : When the heat flow is along curves instead of straight lines,the curves lying in parallel planes the flow is called two dimensional</p> <p>(A.U M/J 2016)</p>	BTL-4	Analyzing	PO1
16	<p>Define steady state condition on heat flow.</p> <p>Solution: Steady state condition in heat flow means that the temp at any point in the body does not vary with time. That is, it is independent of t, the time.</p> <p>(MA2211 A.U M/J 2013)</p>	BTL-1	Remembering	PO1
17	<p>Write the solution of one dimensional heat flow equation , when the time derivative is absent.</p> <p><u>Solution</u> : When time derivative is absent the heat flow equation is $U_{xx} = 0$</p> <p>(A.U N/D 2009, N/D 2015)</p>	BTL-2	Understanding	PO2
18	<p>If the solution of one dimensional heat flow equation depends neither on Fourier cosine series nor on Fourier sine series , what would have been the nature of the end conditions?</p> <p><u>Solution</u> :. One end should be thermally insulated and the other end is at zero temperature. (A.U M/J 2017)</p>	BTL-1	Remembering	PO1
19	<p>State any two laws which are assumed to derive one dimensional heat equation?</p> <p>Solution : (i)The sides of the bar are insulated so that the loss or gain of heat from the sides by conduction or radiation is negligible.</p> <p>(ii)The same amount of heat is applied at all points of the face</p> <p>(A.U M/J 2013)</p>	BTL-1	Remembering	PO1,P O2
20	<p>What are the assumptions made before deriving the one dimensional heat equation?</p> <p>Solution : (i)Heat flows from a higher to lower temperature.</p> <p>(ii)The amount of heat required to produce a given temperature change in a body is proportional to the mass of the body and to the temperature change.</p> <p>(iii)The rate at which heat flows through an area is</p>	BTL-1	Remembering	PO1,P O2

	proportional to the area and to the temperature gradient normal to the area. (A.U M/J 2014)			
21	Write down the two dimensional heat equation both in transient and steady states. Solution : Transient state: $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ Steady state: : $U_{xx} + U_{yy} = 0$ (A.U M/J 2013)			
PART-B				
1	A uniform string is stretched and fastened to two points 'l' apart. Motion is started by displacing the string into the form of the curve $y = kx(l - x)$ and then releasing it from this position at time $t = 0$. Find the displacement of the point of the string at a distance x from one end at time t . (A.U.N/D 2017, N/D 2015,2012, M/J 2013)	BTL-4	Analyzing	PO1,P O2,PO 5
2	A tightly stretched string of length l has its ends fastened at $x = 0$ and $x = l$. The midpoint of the string is then taken to a height h and then released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time.	BTL-4	Analyzing	PO1,P O2,PO 5
3	A tightly stretched string of length $2l$ is fastened at both ends. The midpoint of the string is displaced by a distance 'b' transversely and the string is released from rest in this position. (Find the lateral displacement of a point of the string at time 't' from the instant of release) Find an expression for the transverse displacement of the string at any time during the subsequent motion (A.U. N/D 2011, M/J 2014)	BTL-5	Evaluating	PO1,P O2,PO 5
4	A tightly stretched string of length l is initially at rest in equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find the displacement at any time 't'. (A.U N/D 2014)	BTL-5	Evaluating	PO1,P O2,PO 5
5	A string is stretched between two fixed points at a distance $2l$ apart and the points of the string are given initial	BTL-2	Understanding	PO1,P O2

	<p>velocities v where $v = \begin{cases} \frac{cx}{l} & 0 < x < l \\ \frac{c}{l}(2l - x) & l < x < 2l \end{cases}$, x being the distance from one end point. Find the displacement of the string at any subsequent time. (A.U.N/D 2008).</p>			
6	<p>A rod 30cm long has its ends A and B kept at $20^{\circ}c$ and $80^{\circ}c$ respectively until steady state conditions prevails. The temperature at each end is then suddenly reduced to $0^{\circ}c$ and kept so. Find the resulting temperature function $u(x,t)$ taking $x = 0$ at A. (Nov./Dec. 2009). (A.U.N/D 2011, M/J 2014)</p>	BTL-2	Understanding	PO1,P O2
7	<p>A rod of length l has its ends A and B kept at $0^{\circ}c$ and $120^{\circ}c$ respectively until steady state conditions prevail. If the temperature at B is reduced to $0^{\circ}c$ and so while that of A is maintained, find the temperature distribution of the rod. (A.U M/J 2012,16)</p>	BTL-5	Evaluating	PO1,P O2,PO 5
8	<p>An infinitely long rectangular plate with insulated surface is 10 cm wide. The two long edges and one short edge are kept at zero temperature, while the other short edge $x = 0$ is kept at temperature given by $u = \begin{cases} 20y, & 0 \leq y \leq 5 \\ 20(10 - y), & 5 \leq y \leq 10 \end{cases}$. (A.U M/J 2011,N/D 2012,M/J 2016)</p>	BTL-5	Evaluating	PO1,P O2,PO 5
9	<p>A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance x from one end at time t. (A.U M/J 2015,2016)</p>	BTL-5	Evaluating	PO1,P O2,PO 5
10	<p>A square plate is bounded by the lines $x = 0, x = a, y = 0$ and $y = b$. Its surfaces are insulated and the temperature along $y = b$ is kept at $100^{\circ}C$. Find the steady-state temperature at any point in the plate. [A.U. N/D 2014]</p>	BTL-5	Evaluating	PO1,P O2,PO 5
11	<p>A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. Find</p>	BTL-2	Understanding	PO1,P O2

	the displacement at any time 't'.		
	(A.U N/D 2012)		

UNIT - IV FOURIER TRANSFORM

Fourier integral theorem (without proof) - Fourier transform pair -Sine and Cosine transforms- Properties - Transforms of simple functions - Convolution theorem - Parseval's identity.
 Textbook : Grewal. B.S., and Grewal. J.S., "Numerical Methods in Engineering and Science", 9th Edition, Khanna Publishers, New Delhi, 2007.

PART - A

CO Mapping : C214.2

Q.No	Questions	BT Level	Competence	PO
1	Prove that $F[f(x - a)] = e^{ias} F(s)$ <u>Proof:</u> $F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$ $F(f(x - a)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x - a) e^{isx} dx, \text{ put } t = x - a; dt = dx$ $x \rightarrow \pm\infty \Rightarrow t \rightarrow \pm\infty$ $F(f(x - a)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{is(t+a)} dt = e^{isa} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{ist} dt = e^{isa} F(s).$ (A.U.N/D , N/D 2011,2012, M/J 2013)	BTL-4	Analyzing	PO1
2	Prove that $F(f(x) \cos ax) = \frac{1}{2} [F(s + a) + F(s - a)]$. [MA2211 APR/MAY2011] <u>Proof:</u>	BTL-1	Remembering	PO1, PO2

	$F(f(x)\cos ax) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\cos ax e^{isx} dx$ $= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \frac{e^{iax} + e^{-iax}}{2} e^{isx} dx$ $= \frac{1}{2} \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i(s+a)x} dx + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{i(s-a)x} dx \right)$ $= \frac{1}{2} [F(s+a) + F(s-a)].$ <p>(A.U.N/D 2017, N/D 2015,2012, M/J 2011)</p>			
3	<p>Prove that $F_c(f(x)\sin ax) = \frac{1}{2} [F_s(s+a) + F_s(s-a)]$</p> <p>Proof:</p> $F_c(f(x)\cos ax) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x)\sin ax \cos sx dx$ $= \frac{1}{2} \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) (\sin(s+a)x + \sin(s-a)x) dx$ $= \frac{1}{2} \left(\sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin(s+a)x dx + \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin(s-a)x dx \right)$ $= \frac{1}{2} [F_s(s+a) + F_s(s-a)].$ <p>(A.U M/J 2011,2008)</p>	BTL-2	Understanding	PO1,P O2
4	<p>Find the Fourier sine transform of e^{-x}, $x > 0$.</p> <p>Solution:</p> $F_s(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x)\sin sxdx = \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-x} \sin sxdx$ $= \sqrt{\frac{2}{\pi}} \left[\frac{e^{-x}}{1+s^2} (-\sin sx - s \cos sx) \right]_0^{\infty} = \sqrt{\frac{2}{\pi}} \frac{s}{1+s^2}$ <p>(A.U.N/D 2010)</p>	BTL-4	Analyzing	PO2
5	<p>Write the Fourier transform pair.</p> <p>Proof:</p>	BTL-1	Remembering	PO1

	$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$ $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(s) e^{-isx} ds$ <p>(A.U N/D 2009,N/D 2012,M/J 2015,2016)</p>			
6	<p>Find the Fourier sine transform of $\frac{1}{x}$.</p> <p>Solution:</p> $F_s(f(x)) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sxdx = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{1}{x} \sin sxdx$ $\text{put } sx = \theta; \quad sdx = d\theta; \quad = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \frac{\sin \theta}{\theta} d\theta = \sqrt{\frac{2}{\pi}} \frac{\pi}{2} = \sqrt{\frac{\pi}{2}}.$ <p>(A.U M/J 2009,N/D 2012,M/J 2015,2016)</p>	BTL-2	Understanding	PO1
7	<p>Find the Fourier cosine transform of $f(ax)$.</p> <p>Solution:</p> $F_c(f(ax)) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(ax) \cos sxdx$ $\text{put } t = ax; \quad dt = adx$ $= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(t) \cos\left(\frac{st}{a}\right) \frac{dt}{a} = \frac{1}{a} F_c\left(\frac{s}{a}\right).$ <p>(A.U M/J 2013)</p>	BTL-2	Understanding	PO1,P O2
8	<p>Find the Fourier Cosine transform of e^{-ax}.</p> <p>Solution:</p> $F_c[e^{-ax}] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \cos sxdx = \sqrt{\frac{2}{\pi}} \left[\frac{e^{-ax}}{a^2 + s^2} (-a \cos sx + s \sin sx) \right]_0^{\infty}$ $= \sqrt{\frac{2}{\pi}} \frac{a}{a^2 + s^2}.$ <p>(A.U M/J 2012 N/D 2015,N/D 2009)</p>	BTL-1	Remembering	PO1
9	<p>Find the Fourier transform of $f(x) = \begin{cases} e^{ikx}, & a < x < b \\ 0, & x < a, x > b \end{cases}$</p> <p>Solution:</p>	BTL-1	Remembering	PO1

	$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_a^b e^{ikx} e^{isx} dx = \frac{1}{\sqrt{2\pi}} \int_a^b e^{i(s+k)x} dx = \frac{1}{\sqrt{2\pi}} \left[\frac{e^{i(s+k)x}}{i(s+k)} \right]_a^b$ $= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{i(s+k)b} - e^{i(s+k)a}}{i(s+k)} \right]$ <p>(A.U M/J 2016,N/D 2012,N/D 2009)</p>			
10	<p>State convolution theorem.</p> <p><u>Solution</u> : If F(s) and G(s) are fourier transforms of f(x) and g(x) respectively then the fourier transform of the convolutions of f(x) and g(x) is the product of their fourier transform.</p> <p>(A.U N/D 2012,M/J 2016)</p>	BTL-1	Remembering	PO1
11	<p>Write the Fourier cosine transform pair?</p> <p>Solution :</p> $F_c(s) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} f(x) \cos sxdx$ $f(x) = \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^{\infty} F_c(f(x) \cos sxdx$ <p>(A.U N/D 2011,N/D 2014)</p>	BTL-2	Understanding	PO1,P O2
12	<p>Write Fourier sine transform and its inversion formula?</p> <p>Solution :</p> $F_s(s) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} f(x) \sin sxdx$ $f(x) = \frac{\sqrt{2}}{\sqrt{\pi}} \int_0^{\infty} F_s(f(x) \sin sxdx$	BTL-4	Analyzing	PO1
13	<p>State the modulation theorem in Fourier transform .</p> <p>Solution : If F(s) is the Fourier transform of f(x) , then</p> $F[f(x) \cos ax] = 1/2 [F(s+a) + F(s-a)].$ <p>(A.U.N/D 2014)</p>	BTL-4	Analyzing	PO1,P O2
14	<p>State the Parsevals identity on Fourier transform.</p> <p>Solution : If F(s) is the Fourier transform of f(x), then</p> $\int_{-\infty}^{\infty} f(x) ^2 dx = \int_{-\infty}^{\infty} F(s) ^2 ds$ <p>(A.U N/D 2014,M/J 2016)</p>	BTL-4	Analyzing	PO1
15	<p>State Fourier Integral theorem .</p> <p>Solution : If f(x) is piecewise continuously differentiable & absolutely integrable in $(-\infty, \infty)$ then</p> $f(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) e^{is(x-t)} dt ds$ <p>This is known as Fourier integral theorem</p> <p>(A.U M/J 2014)</p>	BTL-1	Remembering	PO1
16	<p>Define self-reciprocal with respect to Fourier Transform.</p> <p>Solution: If a transformation of a function f(x) is equal to f(s) then the function f(x) is called self-reciprocal</p>	BTL-4	Analyzing	PO1

[A.U. N/D 2013]

PART - B

1	<p>Find the Fourier transform of</p> $f(x) = \begin{cases} a^2 - x^2, & x \leq a \\ 0, & x \phi a \end{cases}$ <p>Hence evaluate</p> $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos\left(\frac{s}{2}\right) dx.$ <p>(A.U M/J 2011,A/M 2012,N/D 2015)</p>	BTL-5	Evaluating	PO1,P O2, PO3,P O5
2	<p>Find the Fourier cosine transform of</p> $f(x) = e^{-ax}, a > 0 \text{ and } g(x) = e^{-bx}, b > 0.$ <p>Hence evaluate $\int_0^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 9)}$.</p> <p>(A.UA/M 2009)</p>	BTL-4	Analyzing	PO1,P O2
3	<p>Find the Fourier Transform of f(x) given by</p> $f(x) = \begin{cases} a - x , & x \leq a \\ 0, & x \phi a \end{cases}$ <p>Hence show that</p> $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2} \text{ and } \int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}.$ <p>(A.U.N/D 2017, N/D 2011,2012, M/J 2013)</p>	BTL-5	Evaluating	PO1,PO2, PO5,PO1 2
4	<p>Find the Fourier transform of</p> $f(x) = \begin{cases} 1, \text{ for } x \leq a \\ 0, \text{ for } x \phi a \phi 0 \end{cases}$ <p>and using Parseval's identity prove that $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$.</p> <p>(A.U.N/D 2017, N/D 2011,2014, M/J 2013)</p>	BTL-4	Analyzing	PO1,PO2, PO5,PO1 2
5	<p>Find the Fourier sine and cosine transform of e^{-ax} and hence find the Fourier sine transform of $\frac{x}{x^2 + a^2}$ and Fourier cosine transform of $\frac{1}{x^2 + a^2}$.</p> <p>(A.U. N/D 2011)</p>	BTL-5	Evaluating	PO1,PO2, PO5,PO1 2
6	<p>Find the Fourier cosine transform of e^{-x^2}.</p> <p>(A.U M/J 2016)</p>	BTL-4	Analyzing	PO1,PO2, PO5,PO1 2
7	<p>Prove that $\frac{1}{\sqrt{x}}$ is self reciprocal under Fourier sine</p>	BTL-5	Evaluating	PO1,PO2, PO5,PO1 2

	and cosine transforms. (A.U N/D 2015,M/J 2014)			
8	Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ using Fourier transforms(MA1201 N/D 2005, M/J 2014,N/D2016)	BTL-5	Evaluating	PO1,PO2, PO5,PO1 2
9	By finding the Fourier cosine transform of $f(x) = e^{-ax} (a \phi 0)$ and using Parseval's identity for cosine transform evaluate $\int_0^{\infty} \frac{dx}{(a^2 + x^2)^2}$. (A.U A/M 2016)	BTL-3	Applying	PO1,PO2, PO5,PO1 2
10	If $F_c(s)$ and $G_c(s)$ are the Fourier cosine transform of $f(x)$ and $g(x)$ respectively, then prove that $\int_0^{\infty} f(x)g(x)dx = \int_0^{\infty} F_c(s)G_c(s)ds$. (A.U N/D 2008,M/J 2015)	BTL-3	Applying	PO1,PO2, PO5,PO1 2
11.	Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2 - x, & \pi \leq x \leq 2\pi \\ 0, & x \geq 2\pi. \end{cases}$ (A.U.N/D 2016, N/D 2011,2012, M/J 2014)	BTL-5	Evaluating	PO1,PO2, PO5,PO1 2
12.	If $F_c(f(x)) = F_c(s)$, prove that $F_c(F_c(x)) = f(s)$. (A.U M/J 2017)	BTL-3	Applying	PO1,PO2, PO5,PO1 2
13	Use transform method to evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$	BTL-3	Applying	PO1,PO2, PO5,PO1 2

UNIT-V Z -TRANSFORMS AND DIFFERENCE EQUATIONS

Z-transforms - Elementary properties - Inverse Z-transform - Convolution theorem -Formation of difference equations - Solution of difference equations using Z-transform.

PART – A

CO Mapping :

Q.No	Questions	BT Level	Competence	PO
1.	Define the unit step sequence. Write its Z- transform. Soln: It is defined as $U(k) : \{1, 1, 1, \dots\} = \begin{cases} 1, & k > 0 \\ 0, & k < 0 \end{cases}$	BTL -1	Remembering	PO1

	Hence $Z[u(k)] = 1 + 1/z + 1/z^2 + \dots = \frac{1}{1-1/z} = \frac{z}{z-1}$ (A.U.N/D 2017, N/D 2010,2012, M/J 2013)			
2.	Form a difference equation by eliminating the arbitrary constant A from $y_n = A.3^n$ Soln: $y_n = A.3^n$, $y_{n+1} = A.3^{n+1} = 3A.3^n = 3y_n$ Hence $y_{n+1} - 3y_n = 0$ (A.U N/D 2010,M/J 2012,2014)	BTL -1	Understanding	PO1
3.	Find the Z transform of $\sin \frac{n\pi}{2}$ Soln: We know that, $z[\sin n\theta] = \frac{z \sin n\theta}{z^2 - 2z \cos \theta + 1}$ Put $\theta = \pi/2$ $z[\sin \frac{n\pi}{2}] = \frac{z \sin n\pi/2}{z^2 - \frac{2z \cos \pi}{2} + 1} = \frac{z}{z^2 + 1}$ (A.U.A/M 2010, M/J 2012)	BTL -5	Understanding	PO1
4.	Find Z(n). Soln: $Z(n) = \frac{z}{(z-1)^2}$ (A.U M/J 2011)	BTL -1	Remembering	PO1
5.	Express $Z\{f(n+1)\}$ in terms of $f(z)$ Soln: $Z\{f(n+1)\} = zf(z) - zf(0)$ (A.U M/J 2011)	BTL -1	Remembering	PO1
6.	Find the value of $z\{f(n)\}$ when $f(n) = na^n$ Soln: $z(na^n) = \frac{az}{(z-a)^2}$ (A.U M/J 2009)	BTL -1	Understanding	PO2,P O5
7.	Find $z[e^{-iat}]$ using Z transform. Soln. By shifting property, $z[e^{-iat}] = z e^{iaT} / z e^{iaT} - 1$ (A.U M/J 2011,N/D 2012)	BTL -1	Remembering	PO1
8.	Find the Z transform of $a^n/n!$. Soln: $z[a^n/n!] = e^{a/z}$ (By definition) (A.U M/J 2012,N/D 2014)	BTL -1	Understanding	PO1
9.	State initial value theorem in Z-transform. Solution : If $f(t) = F(z)$ then $\lim_{t \rightarrow 0} f(t) = \lim_{z \rightarrow \infty} z F(z)$. (A.U.N/D 2015,2013)	BTL -1	Understanding	PO1
10.	State final value theorem in Z-transform. Solution : If $f(t) = F(z)$ then $\lim_{t \rightarrow \infty} f(t) = \lim_{z \rightarrow 0} z F(z)$. State Euler formula. (A.U M/J 2013)	BTL -1	Understanding	PO1
11.	State Convolution theorem on Z-transform. Solution : If $X(z)$ and $Y(z)$ are Z- transforms of $x(n)$ and $y(n)$ respectively then the Z- transform of the convolutions of $x(n)$ and $y(n)$ is the product of their Z- transform. (A.U M/J 2012,N/D 2014)	BTL -1	Understanding	PO1
12.	Define Z-transforms of $f(t)$. Solution : Z-transform for discrete values of t : If $f(t)$ is a	BTL -1	Understanding	PO1

	function defined for discrete values of t where $t=nT$, $n=0,1,2,\dots T$ being the sampling period then $Z\{f(t)\} = F(Z) = \sum_{n=0}^{\infty} f(nT)Z^{-n}$			
13.	Define Z- transform of the sequence. Solution : Let $\{x(n)\}$ be a sequence defined for all integers then its Z-transform is defined to be $Z\{x(n)\} = X(Z) = \sum_{n=0}^{\infty} x(n)Z^{-n}$	BTL -4	Analyzing	PO2
14.	State first shifting theorem. Solution : If $Z\{f(t)\} = F(Z)$ then $Z\{e^{-at} f(t)\} = F(ze^{at})$	BTL -2	Remembering	PO5
15.	Find the Z-Transform of $\cos n\theta$ and $\sin n\theta$? Solution : $Z(\cos n\theta) = \frac{z(z - \cos \theta)}{(z - \cos \theta)^2 + \sin^2 \theta}$ $Z(\sin n\theta) = \frac{z \sin \theta}{(z - \cos \theta)^2 + \sin^2 \theta}$ (A.U N/D 2017)	BTL -2	Remembering	PO5
16.	Find the Z-transform of unit step sequence. Solution: $u(n) = 1$ for $n \geq 0$ $u(n) = 0$ for $n < 0$. Now $Z[u(n)] = \frac{z}{z-1}$	BTL -1	Remembering	PO1
17.	Find the Z-transform of unit sample sequence. Solution: $\delta(n) = 1$ for $n = 0$ $\delta(n) = 0$ for $n > 0$. Now $Z[\delta(n)] = 1$	BTL -1	Understanding	PO1
18.	Form a difference equation by eliminating arbitrary constant from $u_n = a \cdot 2^{n+1}$. Solution : Given , $u_n = a \cdot 2^{n+1}$ $u_{n+1} = a \cdot 2^{n+2}$ Eliminating the constant a, we get $\frac{u_n}{2} = \frac{u_{n+1}}{4}$ We get $2u_n - u_{n+1} = 0$	BTL -1	Understanding	PO1
19.	Form the difference equation from $y_n = a + b \cdot 3^n$ Solution: Given , $y_n = a + b \cdot 3^n$ $y_{n+1} = a + b \cdot 3^{n+1}$ $= a + 3b \cdot 3^n$ $y_{n+2} = a + b \cdot 3^{n+2}$ $= a + 9b \cdot 3^n$	BTL -1	Understanding	PO1

	Eliminating a and b we get, $y_n - 1 = 1$ $y_{n+1} - 1 = 3 = 0$ $y_{n+2} - 1 = 9$ We get $y_{n+2} - 4y_{n+1} + 3y_n = 0$			
20.	Find $Z\left[\frac{a^n}{n!}\right]$ Solution : $Z\left[\frac{a^n}{n!}\right] = e^{\frac{a}{z}}$			

PART-B

1.	Find the Z-transform of $\cos n\theta$ and $\sin n\theta$. Hence deduce the Z-transform of $\cos(n+1)\theta$ and $a^n \sin n\theta$ (A.U.N/D 2012,2015,M/J 2016)	BTL -1	Remembering	PO1,P O2,PO 5
2	Use residue theorem find $Z^{-1}\left[\frac{z(z+1)}{(z-3)^3}\right]$	BTL -3	Applying	PO1,P O2,PO 5
3	Solve $y_{n+2} - 5y_{n+1} + 6y_n = 6^n$, $y_0 = 1$, $y_1 = 0$ (A.U.N/D 2012,2016)	BTL -1	Remembering	PO1,P O2,PO 5
4	Solve using Z-Transform $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$; given $u_0 = u_1 = 0$ [NOV/DEC 2010]	BTL -1	Remembering	PO1,P O2,PO 5
5	Using convolution theorem find the inverse Z transform of $\left(\frac{z}{z-4}\right)^3$ [APRIL/MAY 2010] (A.U.M/J 2010, N/D2014)	BTL -2	Understanding	PO1,P O2,PO 5
6	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = 0$, $y_1 = 0$ (A.U.N/D 2014,2017)	BTL -1	Remembering	PO1,P O2,PO 5
7	Using convolution theorem find $Z^{-1}\left(\frac{z^2}{(z-4)(z-3)}\right)$ (A.U.N/D 2014,2016, M/J2013)	BTL -1	Remembering	PO1,P O2,PO 5
8	Find the inverse Z -transform of $\frac{z^3 - 20z}{(z-2)^3(z-4)}$ (A.U.N/D 2009)	BTL -3	Applying	PO1,P O2,PO 12
9	Find $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z+1)}\right)$ (A.U.N/D 2015,2013)	BTL -3	Applying	PO1,P O2,PO 12

10	State and Prove Convolution theorem (A.U.N/D 2016,2011)	BTL -3	Applying	PO1,P O2,PO 12
11	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = 0$, $y_1 = 0$ (A.U.A/M 2008,2010,N/D2014)	BTL -5	Evaluating	PO1,P O2,PO 5
12	Prove that $Z \left(\frac{1}{n} \right) = \log \left(\frac{z}{z-1} \right)$	BTL -3	Applying	PO1,P O2,PO 12
13	Using convolution theorem evaluate inverse Z- transform of $\left[\frac{z^2}{(z-1)(z-3)} \right]$ (A.U.N/D 2010,2016)	BTL -1	Remembering	PO1,P O2,PO 5